THE DESIGN OF A FABRIC FOR A PRESSURIZED SUIT

BY

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The Design of a Fabric for a Pressurized Suit
Peter Gerald Popper
Submitted to the Department of Mechanical Engineering on January 19, 1959, in partial fulfillment of the requirements for the degree of Master of Science.

ABSTRACT

A study was made to design a fabric structure which can be used in combination with a rubber bladder in a pressurized suit. Several fabrics were developed with the aim of theoretically fulfilling certain requirements of mechanical behavior as set forth by an Air Force specification for space suit design.

From the overall physical requirements of the pressure suit, the stress-strain requirements were calculated for the fabric. This analysis was used to determine qualitatively the type of fabric behavior required and to design quantitatively a series of optimum cloth structures.

Various methods of manufacturing the proposed structures are described.

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January 19, 1959

Professor Alvin Sloane
Secretary of the Faculty
Massachusetts Institute of Technology
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Dear Professor Sloane:

In accordance with the requirements for the degree of Master of Science in Textile Technology, I hereby submit the following thesis entitled, "The Design of a Fabric For a Pressurized Suit".

Respectfully submitted,

Peter Gerald Popper

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ACKNOWLEDGMENT

I would like to express my sincere gratitude to Professor Stanley Backer for his guidance and encouragement throughout this work. It was on his suggestion that I chose this topic for my thesis subject.

I would also like to express my thanks to Mr. Otto Engelhart, Director of Research and Development at Native Lace Corporation for his advice on fabric structures; and to all the members of the M.I.T. staff who worked on this project for their helpful suggestions.

FOREWORD

The problem of designing a fabric for a space suit was first encountered by me in the summer of 1958. At the time I was a research assistant in the Textile Division at M.I.T. working on an Air Force contract for the development of a material to be used in pressure suits. In the fall semester, 1958-59, I returned to a full time student basis and continued to work on selected aspects of the space suit problem as the subject of my thesis.

In my thesis, I have not developed a fabric which meets all the Air Force requirements. However, I have attempted to treat various design techniques which permit theoretical design of materials to approximate the necessary mechanical behavior of pressure suit fabrics as I interpret the problem.

INTRODUCTION

Object

The overall object of this thesis is to design a fabric structure for use in a pressurized suit which will theoretically fulfill certain requirements of mechanical behavior.

More specifically, the object is to:

1. Find the stress-strain relation which is required of a fabric for it to satisfy certain specifications dictated by pressure suit applications.
2. Propose fabric structures which qualitatively have the required stress-strain behavior.
3. Analyze the stress-strain behavior of the fabrics.
4. Determine the bending and torsional rigidity of fabric tubes of different sizes that are under internal pressure.
5. Determine methods by which the designed fabric structures can be produced.

Principle of a Fabric Covered Pressure Suit

A fabric covered pressure suit is constructed with a rubber bladder over the surface of the body to keep the air from leaking out. Over this is a layer of fabric which is used to provide strength. That is, the fabric keeps the rubber from expanding when pressure is applied.

If the modulus of the rubber is low, then the strength properties of the combination rubber and fabric covering can be considered to be essentially the properties of the fabric alone.

The problem of designing a fabric for a pressure suit is that if the fabric is stiff enough to keep the suit dimensionally stable when pressure is applied, it will in general be so stiff as to prevent the wearer from moving freely. If the fabric is one that is easily extensible, then a person will be able to move freely with it on, but the application of pressure will cause it to expand greatly and the suit will no longer fit. Thus, it is necessary to design a structure which will be rigid under the stresses of pressure, and flexible under those of bending and twisting. The fabric must be inextensible up to a certain stress and then easily extensible for any additional stress.

This type of behavior is exactly opposite to that found in most textile fabrics. In general, fabrics are easily extensible up to a certain load. Then, because of jamming they become rigid.

It should be noted that there may be some combination of materials other than a textile fabric over a rubber bladder which can be used in a pressure suit. For example, coated fabrics or plastic laminates might prove successful. Nevertheless, this thesis will only consider fabric covered pressure suits.

Method of Approach in Selecting and Analyzing Fabric Structures

The first step in determining fabric structures was to select elements of structure which had stress-strain curves of the proper shape to fill the bending needs. That is, ones which had a high modulus at first, and then a low one. This was done in two ways:

1. By considering elements under uniaxial stress which are to be put in a fabric where the biaxial stress does not affect the behavior of the elements. (An example of this type of element is the "loop element".)
2. By considering elements under biaxial stress which obtain their behavior because of the biaxial stress. (An example

of this type of element is the "diamond element").

Having the element, the next step was to put it into a fabric structure, and analyze the fabric for its stress-strain curve. When certain parameters were involved that could vary the fabric behavior, the analysis either determined what value they must have for the fabric to meet the requirements, or what value they must have for the fabric to reach its optimum condition.

The next step was to consider shearing of the fabric. The shear stress necessary for the fabric to reach the required shear strain was calculated for various methods by which deformation could occur.

The materials from which the structures were designed were either Hookian, inelastic, or combinations of the two.

General Remarks

The analysis which is done in this thesis does not utilize any difficult concepts of either mathematics or mechanics. It does however make use of relatively elementary principles of statics and strength of materials to obtain the stress-strain relations in structures of complicated geometry.

In many instances it would probably have been possible to use more exact methods of analysis than those which I have used. However, this would have been wasted effort. The effects of variability, visco-elastic behavior, and friction associated with textile materials are so great that theoretical analysis can be used only as a starting point for experimental work. That is exactly the aim of this thesis.

AIR FORCE SPECIFICATIONS

The performance requirements specified for pressure suit fabrics are as follows:

A cylinder of three inch radius and twenty inch length under ten psi internal pressure shall meet the following requirements:

90° bend, min.	-	5 lbs max force
90° twist, min.	-	5 lbs max force
Elongation	-	2.0% max.
Circumference increase	-	6.0% max.

Several other specifications were given in regard to the fabric for the pressure suit. They will not be considered in this thesis since they do not concern mechanical behavior of the pressurized cylinder.

REDEFINITION OF THE PHYSICAL REQUIREMENTS

Introduction

In order to design fabric structure for the pressure suit, I have found it convenient to redefine some of the physical requirements of the end item or its components, into common engineering terms. Certain assumptions were made in doing this, and they will be explained.

Bending Requirements

The specification for bending gives a minimum load that must be applied to a cylinder under internal pressure to produce a 90° bend. The dimensions of the cylinder are given. This specification was redefined for two reasons:

1. A 90° bend can be obtained by several ways of deformation. To be precise the radius of curvature, or the curvature of the bend must be given.
2. The loading requirement would be better expressed as a moment rather than a force. This is because a moment produces a stress which does not vary along the length of the cylinder.

A good specification for bending is one which gives the minimum allowable moment which has to be applied to produce a given curvature. I have rewritten the space suit specification in this form.

The moment was taken to be their force times the length of the cylinder. (This is an assumption, and it is actually the greatest moment occurring in their system.)

$$\begin{aligned} M &= FL \\ &= (5)(20) \\ &= 100 \text{ lb in} \end{aligned}$$

The curvature was taken to be the curvature which is produced when a 20 inch length is put into a 90° arc. This is an assumption since the specification does not state how much of the length is part of the arc. Since the arc length is the radius times the arc angle in radians:

$$\left(\frac{1}{K}\right)\frac{\pi}{2} = 20$$

$$K = 0.785 \text{ in}^{-1}$$

The resulting specification for bending reads: In order to produce a curvature of 0.785 inches^{-1} on a 3 inch radius cylinder under internal pressure of 10 psi, the maximum allowable moment is 100 in-lb.

Twisting Requirements

The specification for twisting gives the maximum allowable force to produce a twist of 90°. This specification was redefined for two reasons:

1. It does not state where or how the force is applied to the cylinder to produce twisting. A twisting moment should be given.
2. Although a twist of 90° is perfectly clear, it is more convenient to give the radians per length of twist, - the torsion.

A good way for this specification to read is to give the maximum allowable twisting moment which can be applied to produce a given torsion. I have put the specification into this form.

By assuming that the force is applied at the outside of the cylinder, the twisting moment is found to be:

$$M = FR$$
$$= (5)(3)$$
$$= 15 \text{ lb in}$$

The torsion is found by rewriting 90° twist per 20 inches of length to be:

$$\tau = \frac{\pi/2}{20}$$
$$= 0.785 \text{ rad/in}$$

The resulting specification for twisting reads: In order to produce a torsion of 0.875 inches^{-1}, on a three inch radius cylinder under 10 psi internal pressure, the maximum allowable twisting moment is 15 in-lb.

General Remarks

The requirements for strain due to pressure as given by the specifications was not changed.

In redefining the physical requirements, I have to make certain assumptions in determining just what was intended by the Air Force. In doing this, I have taken what I feel to be the most logical interpretation. It is also the one which gives the least difficult requirements.

A list of redefined specifications can be found in the Results on page 87 .

FABRIC STRESS-STRAIN REQUIREMENTS

In order to design a fabric with certain mechanical properties, it is necessary to determine what stress-strain relations are required. That will be done in this section.

By changing the overall fabric requirements into stress-strain requirements, the radius of the tube and its length both disappear from the equations.

Longitudinal Stress-Strain Relation

At pressure, the longitudinal fabric stress of a thin walled cylinder is:

$$S_{yp} = \frac{PR}{2} = \frac{(10)(3)}{2} = 15 \text{ lbs/in}$$

Where:

S_{yp} = stress in longitudinal direction due to pressure

P = pressure

R = radius of cylinder

The maximum allowable strain at pressure in this direction is:

$$\varepsilon_{YP} = .02$$

Thus, the maximum allowable modulus of pressurization can be found to be:

$$E_p = \frac{15}{.02} = 750 \text{ lbs/in}$$

When bending takes place, a tensile strain is imposed on the top of the cylinder, and

$$\mathcal{E}_{Yb} = KR$$
$$= (.0785)(3) = \underline{.236}$$

Where:

$\mathcal{E}_{Yb}$ = max. strain of bending

K = curvature

Knowing the allowable moment to produce the given curvature, we can calculate the allowable stress with the use of the simple beam equation for thin walled cylinders.

$$S_{Yb} = \frac{M_b}{\pi R^2}$$
$$= \frac{100}{\pi 9}$$
$$= 3.53 \text{ lbs/in}$$

Where:

M_b = allowable bending moment

S_{Yb} = allowable bending stress

From this, the maximum allowable modulus of bending is found to be:

$$E_b = \frac{3.53}{.236} = 15.0 \text{ lbs/in}$$

The resulting stress-strain curve is plotted on Figure 24 . We can see from it just how stiff the fabric must be for pressurzing, and how easily extensible it must be for bending.

It should be noted that this analysis is only approximate since it is based on all the assumptions of the beam theory.

Circumferential Stress-Strain Relation

Using a similar approach as in the previous section, we can find the requirements for pressurization. Since bending does not impose any strains of importance in this direction, there are no bending requirements. The only requirements on these directions are that up to 30 lbs/inch stress, the strain must be less than 6%.

Shear Stress-Strain Relation

Since there is no shear stress (in xy direction) imposed at pressurization, the only shear requirements are derived from the twisting specification.

At the torsion required, the shear strain can be found as:

$$\varepsilon_s = \tau R$$
$$= (.0785)(3)$$
$$= .236$$

Where

ε_s = shear strain

τ = torsion

Knowing the allowable twisting moment, the allowable shear stress can be calculated from the simple torsion equation of thin walled cylinders.

$$S_s = \frac{M_T}{2\pi R^2}$$
$$= \frac{15}{2\pi 9}$$
$$= .215 \text{ lbs/in}$$

Where:

S_s = shear stress

M_T = twisting moment

G = allowable shear modulus

ANALYSIS OF DIAMOND FABRIC

The method of selecting fabrics as outlined in the introduction was to propose fabric elements which qualitatively have the stress-strain behavior which is required in the longitudinal direction of the cylinder. A simple diamond shape arrangement of four yarns has that behavior. This is not at all obvious since it is caused by the effect of the biaxial stress condition.

It will be shown that the strain of this element is a function of the ratio of forces acting on it, and not their magnitudes. During pressurizing, this ratio is constant and no strain will take place provided the proper initial geometry is used. When the ratio increases, as in bending, a strain will result.

After having determined what the best possible stress-strain this fabric can have for bending, the problem of shearing will be discussed and analyzed.

Bending Stress-Strain

To determine the stress-strain equation of this fabric, we must first consider the unit of structure.

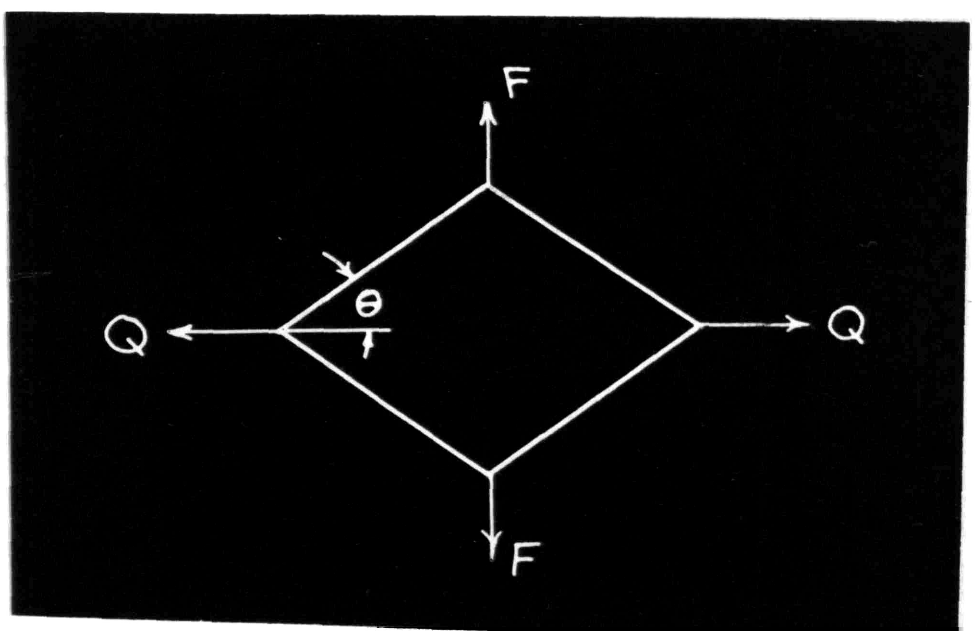


FIGURE 20.

From elementary statics, we can find the relation between the forces.

$$\frac{F}{Q} = \tan\theta$$

It is also possible to find the relation between the angle θ at a strain of $\mathcal{E}_y$ and the original angle θ_o to be:

$$\sin\theta = \sin\theta_o\,(1+\mathcal{E}_y)$$

Where:

$\mathcal{E}_y$ = strain in vertical ("y") direction

θ_o = angle at zero strain

From trigonometric relations:

$$\tan\theta = \frac{\sin\theta_o\,(1+\mathcal{E}_y)}{\sqrt{1-\sin^2\theta_o\,(1+\mathcal{E}_y)^2}}$$

The next step is to determine what the stress-strain behavior of a fabric made up of many of these elements side by side and above each other. If we let:

S_y = stress in the y-direction (lbs/in)

S_x = stress in the x-direction

P_1 = spacing between y-wise elements

P_2 = spacing between x-wise elements

It is easily seen that:

$$S_y = \frac{F}{P_1}$$

$$S_x = \frac{Q}{P_2}$$

From the fabric geometry the spacing ratio can be found, and the previous equations substituted.

$$\frac{P_2}{P_1} = \tan\theta$$

$$\frac{S_y}{S_x} = \frac{F}{Q}\,\frac{P_2}{P_1}$$

$$= \tan^2\theta$$

$$\frac{S_y}{S_x} = \frac{\sin^2\theta_o(1+\mathcal{E}_y)^2}{1-\sin^2\theta_o(1+\mathcal{E}_y)^2}$$

This equation is the stress-strain relation of the diamond fabric. To get the proper initial conditions, we set the strain equal to zero when the stress ratio is that of pressurizing.

$$\left(\frac{S_y}{S_x}\right)_{Press} = \frac{1}{2}$$

$$\mathcal{E}_y = 0$$

$$\frac{1}{2} = \frac{\sin^2\theta_o (1+0)^2}{1-\sin^2\theta_o (1+0)^2}$$

$$\tan^2\theta_o = \frac{1}{2}$$

$$\theta_o = 35.3^\circ$$

This is the required initial angle for no strain at pressurizing. Substituting this into the stress equation, we get:

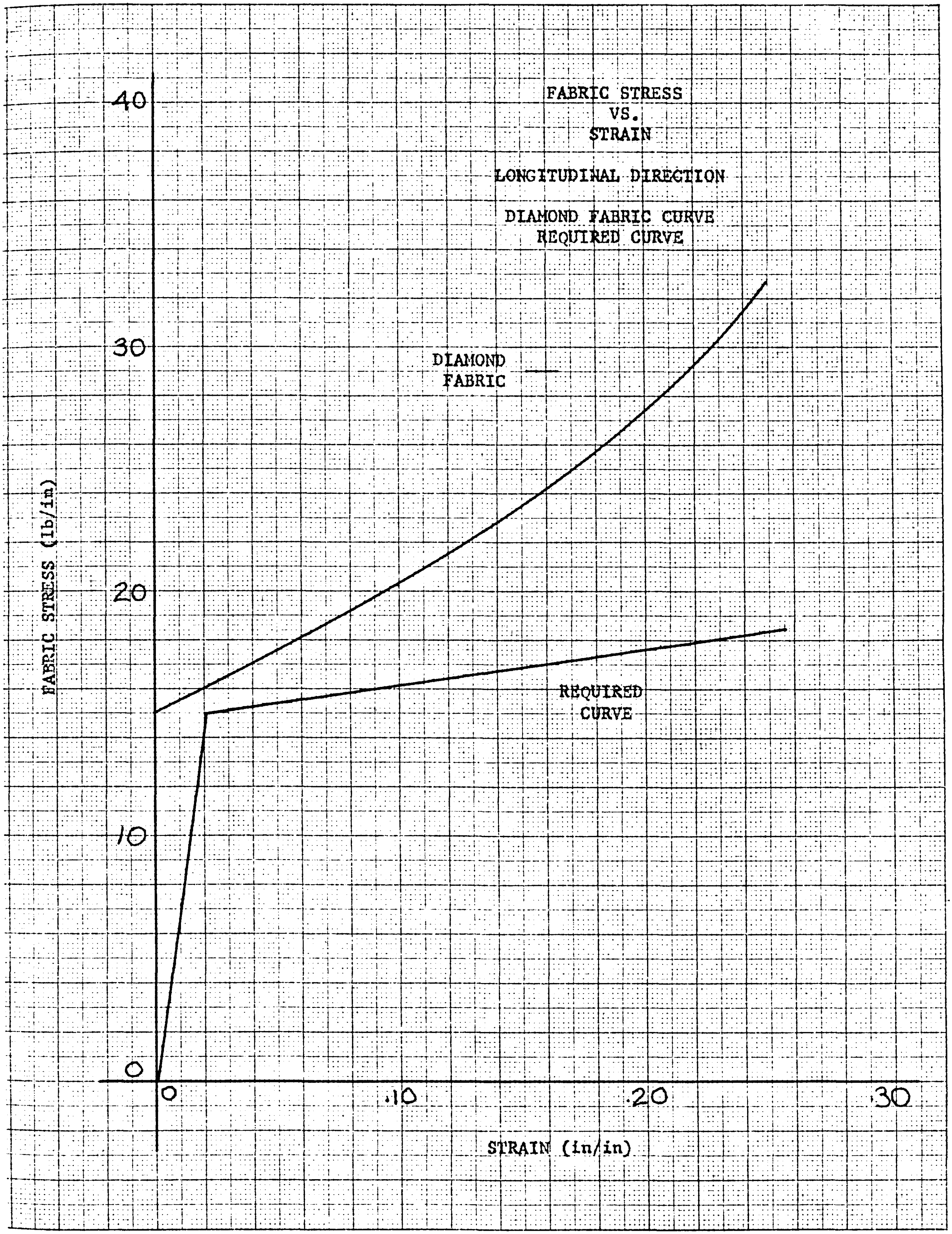
$$\frac{S_y}{S_x} = \frac{(1+\mathcal{E}_y)^2}{3-(1+\mathcal{E}_y)^2}$$

The stress S_x is almost constant in bending, while S_y increases. Substituting the value of constant stress, we find the stress-strain equation of the fabric to be:

$$S_y = \frac{30\,(1+\mathcal{E}_y)^2}{3-(1+\mathcal{E}_y)^2}$$

Thus, we can now plot the stress-strain relation of the diamond fabric in the y-direction for constant stress ratio up to pressure, and then constant stress in the x-direction. This is done in Figure 24. along with the required stress-strain curve.

It should be noted that the initial angle can be made to allow 2% extension at pressure. This gives slightly better results, - 30 lb/in at 25% strain. Since the allowable strain at pressure will probably come from yarn extension, it would not be advisable to use this method.


FABRIC STRESS
VS.
STRAIN
LONGITUDINAL DIRECTION
DIAMOND FABRIC CURVE
REQUIRED CURVE
DIAMOND FABRIC
REQUIRED CURVE
FABRIC STRESS (lb/in)
40
30
20
10
0
0
.10
.20
.30
STRAIN (in/in)

Shearing of Diamond Fabric

In order for a fabric of the diamond geometry to deform by a shear stress, one of two things must happen; either the yarns buckle, or slipping takes place at the yarn intersections. The shear stress necessary for deformation by both these methods will be determined in the following sections.

Shear Deformation by Buckling - (Diamond Fabric with Rigid Intersections)

The diamond, when not under pressure, will behave as any other textile fabric and have little or no resistance to a pure shear stress. This is because most textile yarns cannot resist compression and they buckle.

However, this shear mobility does not exist in a fabric under pressure. This is because pressure causes tensile stresses.

The yarns must have zero tension before they will buckle, and therefore a shear stress will have to remove all the tension produced by pressure before buckling will occur. The object of this analysis is to determine how much shear stress is necessary to do this.

Considering a diamond fabric with rigid intersections, we can find an expression for yarn tensions during the combined loading of pressure and shear stress.

Let:

S_s = shear stress applied

S = shear force in x-direction

S' = shear force in y-direction

T_1 = tension in yarn #1

T_2 = tension in yarn #2

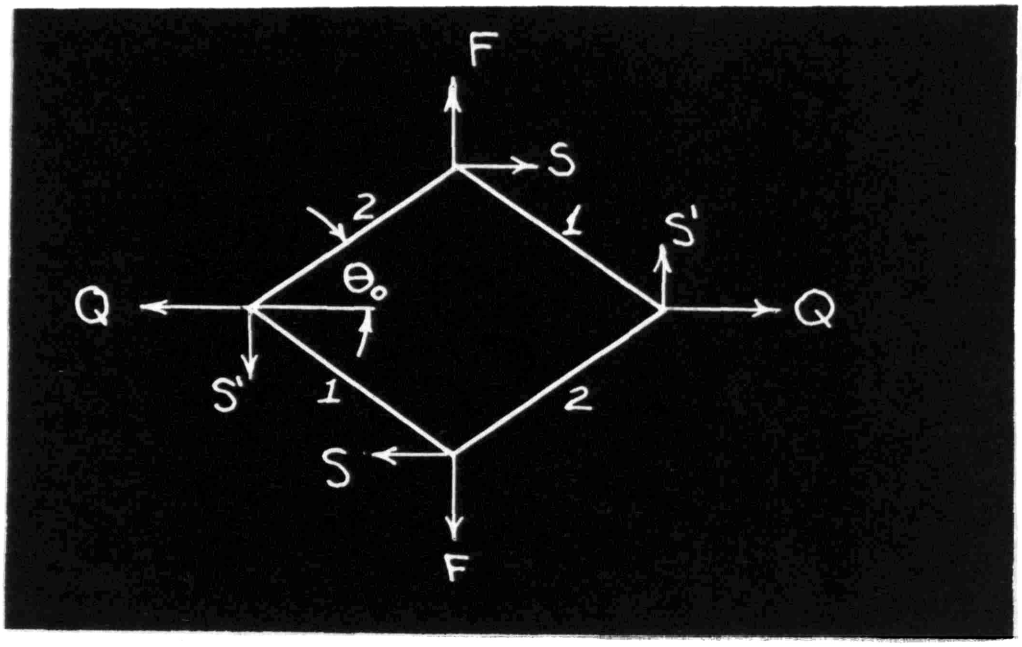


FIGURE 26.

If the fabric is designed to resist the pressure stresses with zero strain ($\Theta_0 = 35.3°$) the relations between F and Q, and S' and S can be found.

$$Q = F\sqrt{2}$$

$$S' = \frac{S}{\sqrt{2}}$$

From the conditions of equilibrium, we can find the equations for the tension in the yarns.

$$F \sin\Theta_0 - S\cos\Theta_0 - T_1 - T_2 \cos\left[2(90-\Theta_0)\right] = 0$$

$$F\sqrt{2}\cos\Theta_0 + T_2 \cos 2(90-\Theta_0) - T_1 - \frac{S}{\sqrt{2}}\sin\Theta_0 = 0$$

Yarn #1 will have the smaller tension and will be first to buckle. Therefore, only T_1 is of interest to us. It can be found by solving the above equations.

$$T_1 = \frac{F}{2}\left(\sin\theta_0 + \sqrt{2}\cos\theta_0\right) - \frac{S}{2}\left(\cos\theta_0 + \frac{\sin\theta_0}{\sqrt{2}}\right)$$

Setting this equal to zero, we can find the condition necessary for buckling.

$$\frac{F}{S} = \frac{\cos\theta_0 + \frac{\sin\theta_0}{\sqrt{2}}}{\sin\theta_0 + \sqrt{2}\cos\theta_0}$$

This can be substituted in the relation for stresses.

$$S_Y = \frac{F}{P_1}$$

$$S_S = \frac{S}{P_1}$$

$$\frac{S_S}{S_Y} = \frac{\sqrt{2}\sin\theta_0 + 2\cos\theta_0}{\sqrt{2}\cos\theta_0 + \sin\theta_0}$$

Solving for shear stress with s_Y equal to 15 lb/in, we get:

$$S_S = 21.3 \text{ lbs/in}$$

This is the shear stress necessary for buckling when the tensile stress of pressure acts. It is considerably higher than the allowable shear stress which is 0.215 lb/in.

It is doubtful if the diamond fabric could deform in shear by buckling within the requirements. This analysis was presented nevertheless, since the result obtained here is not at all obvious and because the only way a rigid intersection diamond fabric can deform in shear is by buckling.

Shear Deformation by Slipping - (Diamond Fabric with Slipping at Intersections)

When the diamond fabric has both shear and tensile stresses on it, there will be a tendency for the yarns to slip through their intersections. When this happens, the diamond shapes will deform into parallelograms. This is shown on Figure 80/b.

In this analysis, we will consider a fabric with intersections similar to those of the bobbinet fabric (Figure 80/a), and determine how much stress is necessary to hold the fabric in equilibrium at a given strain.

Let:

μ = coefficient of friction

ε_s = shear strain

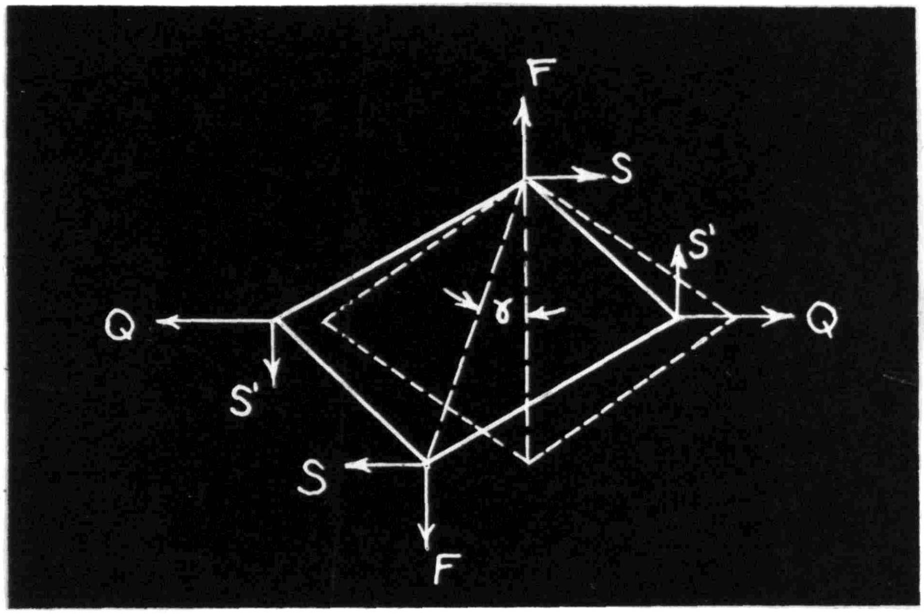


FIGURE 29.

The forces acting on this element are similar to those of Figure 29 . Here, the intersections are not rigid, and the resisting force S' is produced by the effect of friction. The normal force acting upon the intersection is Q.

From conditions of equilibrium, we can now find the equation for shear stress as a function of frictional coefficient, tensile stress, and shear strain.

From the definition of shear strain:

$$\varepsilon_s = \tan \gamma$$

From the definition of coefficient of friction:

$$\mu = \frac{S'}{Q}$$

Considering equilibrium conditions and the proper initial angle:

$$Q = \sqrt{2} F$$

$$\Sigma M_o = 0$$

$$S' = \frac{1}{\sqrt{2}}\left(S - F \tan\gamma\right)$$

$$\mu\sqrt{2}F = \frac{1}{\sqrt{2}}\left(S - F\,\mathcal{E}_s\right)$$

$$S = F\mathcal{E}_s + 2\mu F$$

$$S_s = S_y\mathcal{E}_s + 2\mu S_y$$

$$S_s = 15\,\mathcal{E}_s + 30\mu$$

This is the required equation. It can be seen that if the coefficient of friction is very small, the equation can be approximated by:

$$S_s = 15\,\mathcal{E}_s$$

At the required strain of 0.236, the stress will be 3.54 lb/in rather than the allowable stress of 0.215 lb/in.

It is interesting to note that this fabric has the same properties if it is rotated 90°. (Not derived here.) Also, it is interesting to see that even with a zero coefficient of friction, there will still be a strong resistance to shear deformation, in this fabric.

Shear Stress During Bending and Twisting

Although the space suit specifications only consider bending and twisting taking place separately, it is obvious that some parts of the body will bend and twist simultaneously.

Using an approach similar to that in the previous section, the general shear stress equation was found to be:

$$S_s = S_y\,\mathcal{E}_s + S_x\mu$$

In bending, s_y will vary with the distance from the neutral axis. This will cause s_s to vary. Since the net increase of s_y is close to zero in bending, the net change of s_s will be close to zero. Thus, the moments calculated for bending and twisting will not be changed if both loadings are applied simultaneously.

MODIFIED DIAMOND FABRICS

In the previous section, the diamond fabric was analyzed to determine how close to the specifications it could be designed. Qualitatively the fabric has the proper behavior, but numerically not. Now, I will go through the analysis of several modified diamond fabrics to determine whether it is possible to design a satisfactory fabric with this general type of structure; and, if so, what the special requirements.

Three modifications will be considered. They include:

1. Diamond Fabric with Compression Resistant Yarn
2. Diamond Fabric with Elastic Yarns
3. Diamond Fabric with Series Yarns

DIAMOND FABRIC WITH COMPRESSION RESISTANT YARN

Bending

In the analysis of the diamond fabric, the initial angle was set to give zero strain at pressurization. It was possible to get slightly better results by setting this angle. The lower it is, the lower will be the stress at the required strain. However, if the angle is too low, the pressure strains will be too large. This particular modification of the diamond fabric permits the lowering of the initial angle without affecting the pressure requirements.

The fabric consists of diamond elements as before, but this time each one has a yarn running through its center. These yarns run in the circumferential or x-direction. (See Figure 81/b), and they must be rigidly fixed at each intersection. Also, these yarns must be capable of resisting some compressive stress before buckling.

We will now proceed to analyze this fabric and determine just how much compressive resistance the yarns must have in order to make the fabric able to meet the bending requirements.

First we assume a stress-strain curve of compression for the yarns. At the initial application of load, the yarns will remain rigid, and then, after a critical load is reached, buckling takes place. A large deformation will occur, and the only resistance to further displacement will be due to the yarn bending rigidity which is very small.

The stress-strain curve will have a very high initial slope, and then after buckling, a decreasing stress that goes close to zero.

We will now determine how high the buckling load of the yarns must be. Using the same method as in the regular diamond fabric, we find the relation between the forces acting on any element. The compression yarn must not buckle until after the fabric is completely pressurized. Just at pressurization we find (Using same notation as before.)

$$\frac{F}{Q+Q_1} = \tan\theta_0$$

Where

$Q' =$ buckling force of yarn (lb)

From the definition of fabric stress, we find that just at pressurization:

$$\frac{S_y}{S_x} = \left(\frac{F}{P_{1_0}}\right)\left(\frac{P_{2_0}}{Q}\right)$$

$$= \frac{F}{Q}\tan\theta_0$$

Substituting and solving for Q', we find:

$$\frac{S_y}{S_x} = \tan^2\theta_0\left[1+\frac{Q'}{Q}\right]$$

$$Q' = \frac{S_y}{S_x}\,\frac{Q}{\tan^2\theta_0} - Q$$

Since after buckling, the "compression" yarn will have no more resistance, the fabric will behave just like a regular diamond fabric. We will now determine just how much the initial angle of the diamond fabric must be changed in order for it to meet the requirements. That is, find which θ_0 will give 18.5 lb/in at 25% strain.

$$\frac{S_y}{S_x} = \frac{\sin^2\theta_o(1+\varepsilon)^2}{1-\sin^2\theta_o(1+\varepsilon)^2}$$

$$\frac{18.5}{30} = \frac{\sin^2\theta_o(1.25)^2}{1-\sin^2\theta_o(1.25)^2}$$

$$\theta_o = 29.7°$$

$$\tan^2\theta_o = .325$$

Knowing what initial angle is necessary, we substitute it into the equation for Q' and find the required buckling load.

$$Q' = \frac{S_y}{S_x}\,\frac{Q}{(.325)} - Q$$

$$\frac{S_y}{S_x} = \frac{1}{2}$$

$$Q = S_x P_{2_o}$$

$$= 30\,P_{2_o}$$

$$\frac{Q'}{P_{2_o}} = .54 \text{ lb/in}$$

This equation shows that the required buckling load divided by the initial spacing of the yarns must be .54 lb/in.

If a yarn can be constructed to give this buckling load, the diamond fabric will meet the bending requirements. Whether or not this can be done is a question which must be answered by practical considerations.

Twisting

In order for the yarns of this fabric to resist compression, they must be held fixed in the intersections. That means that this fabric can only deform in shear by buckling of the diamond yarns. This was analyzed in the section on Shearing of Diamond Fabric with Rigid Intersections. The required shear stress for deformation was found to be 22.3 lb/in.

DIAMOND FABRIC WITH ELASTIC YARNS

Bending

In the analysis of the diamond fabric, an assumption was made that the yarns are completely inextensible, In this analysis we will not make that assumption, but will only approximate the yarn behavior to be Hookian.

By selecting the proper yarn modulus, and proper fabric geometry, it is possible to obtain just the allowable strain at pressurization. Then, if the yarns retain their initial modulus, they will extend further due to the stresses of bending. The total extension of any diamond elements will now be due to two things:

1. Change of angle of the diamond.
2. Extension of the yarns.

To get the maximum extension in bending, the initial angle will again be set at 35° , so that yarns with the lowest possible modulus can be used, to give the allowable 2% pressure strain. The lower the modulus, the greater the bending strains will be at any stress level.

The object of this analysis to to determine the stress-strain curve of this fabric and to find the stress at a strain of 25%. Also, the yarns requirements which give a fabric strain of 2% at pressure will be found.

As before, we consider the element of structure and determine the relation between the forces. Using the same notation as before, we find:

$$\frac{F}{Q} = \tan\theta \qquad (1)$$

To find the relation between θ and strain, we must take the yarn extension into consideration. Since the yarn extension is dependent on tension, the force must enter into the equation. The result is

$$\sin\theta = (\varepsilon+1)\sin\theta_o - \frac{F}{2AE} \tag{2}$$

Where

A = yarn area

E = yarn modulus

This equation differs from the regular diamond equation by a term of -F/2AE.

Considering a fabric made of the diamond elements, we again find that:

$$S_y = \frac{F}{P_1}$$

$$S_x = \frac{Q}{P_2}$$

$$\frac{S_y}{S_x} = \tan^2\theta \tag{3}$$

Since

$$\varepsilon = \frac{P_2}{P_{2o}} - 1$$

and,

$$\frac{P_2}{P_1} = \tan\theta$$

$$F = S_y P_1$$

$$Q = S_x P_2$$

We obtain expression for F:

$$F = S_y P_2 \cot\theta$$

$$F = S_y \cot\theta(\mathcal{E}+1)P_{2_0} \qquad (4)$$

Reviewing the equations, we find that we have: (Referring to the equation numbers)

(1) $F = F(Q,\theta)$

(2) $\theta = \theta(\mathcal{E},F)$

(3) $S_y = S_y(S_x,\theta)$

(4) $F = F(S_y,\theta,\mathcal{E})$

These equations are four independent relations between six variables. It is possible to solve for any one as a function of two of the others. This was done to find the relation between stresses and strain.

The algebra involved in solving these equations is very long, and will not be presented here. The resulting equation is:

$$\frac{S_y}{S_x} = \frac{J^2}{1-J^2}$$

$$J = (\sin 2\theta_0)(1+\mathcal{E})\left(1 - \frac{\sqrt{S_x S_y}}{\frac{EA}{W}}\right)$$

Where:

W = length of one side of diamond

We find that the strain depends on both the stress ratio and the stress product in this case. It also depends on EA/W of the yarn, which is its spring constant.

Setting in the initial conditions,

$$\frac{S_y}{S_x} = \frac{1}{2}$$

$$\mathcal{E} = .02$$

$$\theta_o = 35^\circ$$

we can find the required spring constant of the yarn to be:

$$\frac{EA}{W} = 1{,}140 \text{ lbs/in}$$

Now we have the complete stress-strain equation which is valid throughout bending.

$$S_y = \frac{30\, J^2}{1 - J^2}$$

$$J = .333(1+\mathcal{E})(1 - .0263\sqrt{S_y})$$

When the strain is 25%, the stress is:

$$S_{y_{25\%}} = 28.8 \text{ lbs/in}$$

This is a considerable improvement over the regular diamond fabric.

Twisting

This fabric can be made with either fixed or slipping intersections, and the shear stress will be the same as previously determined for the diamond fabric. There is a possibility that extension of the yarns will be some aid in shearing, but because of the high spring constant which is necessary, it is doubtful.

DIAMOND FABRIC WITH ELASTIC YARNS IN SERIES

Introduction

Another method of modifying the diamond fabric is to insert yarns in series with the diamond elements. The fabric will then be composed of diamond elements, each with a length of straight yarn running in the longitudinal direction.

The function of the straight yarns is to permit the fabric to deform in shear. The longer these yarns are, the easier this will be, but, since the fractional length of the diamond elements will be smaller, it will be more difficult to stretch the complete element to a given strain.

The object of this analysis is to determine just how the insertion of these series yarns will affect both the bending and shearing properties of the diamond fabric.

Bending

In designing this fabric, we have a choice of making the yarns rigid or elastic. It can be shown that the best results are obtained by making the yarns in the diamond element inextensible, and those in series position Hookian.

The diamond element will be designed to give zero strain at the stresses of pressurization. The series yarns, however, will be designed so that their modulus permits just the allowable strain at pressurization.

In this analysis, strain will be found as a function of stress because of less involved algebraic equations. We start by writing:

$$\varepsilon_T = D\varepsilon_D + (1-D)\varepsilon_L$$

Where

ε_D = strain of the diamond element

D = fractional length of the diamond element

ε_L = strain of series yarn

ε_T = total strain of combined element

From the stress-strain equation of the diamond fabric we can solve for strain:

$$\frac{S_y}{S_x} = \frac{(1+\varepsilon_D)^2}{3-(1+\varepsilon_D)^2}$$

$$\varepsilon_D = 1.73\sqrt{\frac{S_y}{S_x+S_y}} - 1$$

The strain of the series yarns will be:

$$\varepsilon_L = \frac{F}{AE}$$

$$= \frac{S_y P_1}{AE}$$

Where:

F = force in series yarn (lb)

S_y = fabric stress in longitudinal or y-direction (lb/in)

A = area of series yarns (in^2)

E = modulus of series yarns (lb/in^2)

P_1 = spacing between y-wise elements (in)

Since the diamond element contracts in the x-direction as it extends in the y-direction, the series yarns will come closer together during strain. Thus, their spacing will decrease, and :

$$P_1 = P_2 \cot\theta$$

$$P_2 = P_{2_0}(1+\varepsilon_D)$$

Where:

P_2 = y-wise length of diamond element (in)

P_{2_0} = initial value of above (in)

θ = diamond angle

Taking the relation for θ from the diamond fabric analysis and making the necessary substitutions, we can find the relation between the strain of the series yarns and the stresses.

$$\cot\theta = \frac{\sqrt{1-.333(1+\varepsilon_D)^2}}{.577(1+\varepsilon_D)}$$

$$\varepsilon_L = \frac{S_y P_2}{AE} \frac{\sqrt{1-.333(1+\varepsilon_D)^2}}{.577}$$

$$.333(1+\varepsilon_D)^2 = \frac{S_y}{S_y+S_x}$$

$$\varepsilon_L = \frac{S_y P_{2_0}}{AE(.577)} \sqrt{\frac{S_x}{S_x+S_y}}$$

Substituting this into the original equation for strain, together with the strain of the diamond element, we find:

$$\varepsilon_T = D\left[1.73\sqrt{\frac{S_y}{S_y+S_x}} - 1\right] + (1-D)\left[\frac{S_y P_{2_0}}{AE}(1.73)\sqrt{\frac{S_x}{S_y+S_x}}\right]$$

By setting the strain of the series yarns equal to the allowable strain at pressurization, we find the requirements of physical properties and geometry for optimum behavior.

$$.02 = (1-D)\,\frac{P_{2_0}}{AE}\,(1.73)\sqrt{\frac{30}{45}}$$

$$\frac{AE}{P_{2_0}} = (1-D)\,1060$$

Substituting this into the equation for total strain, we get the desired relation. This is plotted on Figure 48. for various values of D.

$$\varepsilon_T = D\left[1.73\sqrt{\frac{S_y}{S_y+30}} - 1\right] + (1-D)\left[.00164\,S_y\sqrt{\frac{30}{30+S_y}}\right]$$

As expected, the smaller the fractional length of the diamond element is, the further the fabric will be from meeting the bending stress-strain requirements.

Shear Stress-Strain

The shear stress-strain relation cannot be determined exactly because of many involved factors which enter into the equation. However, an approximate analysis can be easily obtained.

The assumption that must be made is that there is no increase in shear stress due to restriction from contraction of the fabric cylinder as it is twisted. This means that the fabric tube can be considered to behave as if it were free to contract in the longitudinal direction. (Actually, it will not be free to do so, but the diamond elements will extend and the series yarns will stretch to give this effect.)

We first consider a cylinder under pressure which is made of nothing but yarns running in the longitudinal direction. If we assume that the yarns have no bending or twisting resistance, we can find the shear stress necessary to produce a displacement of the yarns using the conditions of equilibrium. The forces acting on any element will be a pressure force and a shearing force. The yarn will not be able to resist bending or twisting so it will line up with the resultant of these two forces. The angle of rotation of the yarn in doing this will be:

$$\tan \gamma = \frac{S}{F}$$

But, this is exactly the shear strain of the yarns by definition. Converting to fabric stresses, we get:

$$\varepsilon_s = \frac{S_s}{S_y}$$

Where:

γ = angle of yarn displacement (radians)

S = shear force on the yarn (lb)

F = longitudinal force on the yarn (lb)

ε_s = shear strain (in/in)

S_y = stress in y-direction (lb/in)

S_s = shear stress (lb/in)

In order to consider the behavior of a fabric composed of a combination of straight yarns and other elements, we must consider the fractional length of the straight yarns. As before, the fractional length of the straight yarns will be (D-1).

Where

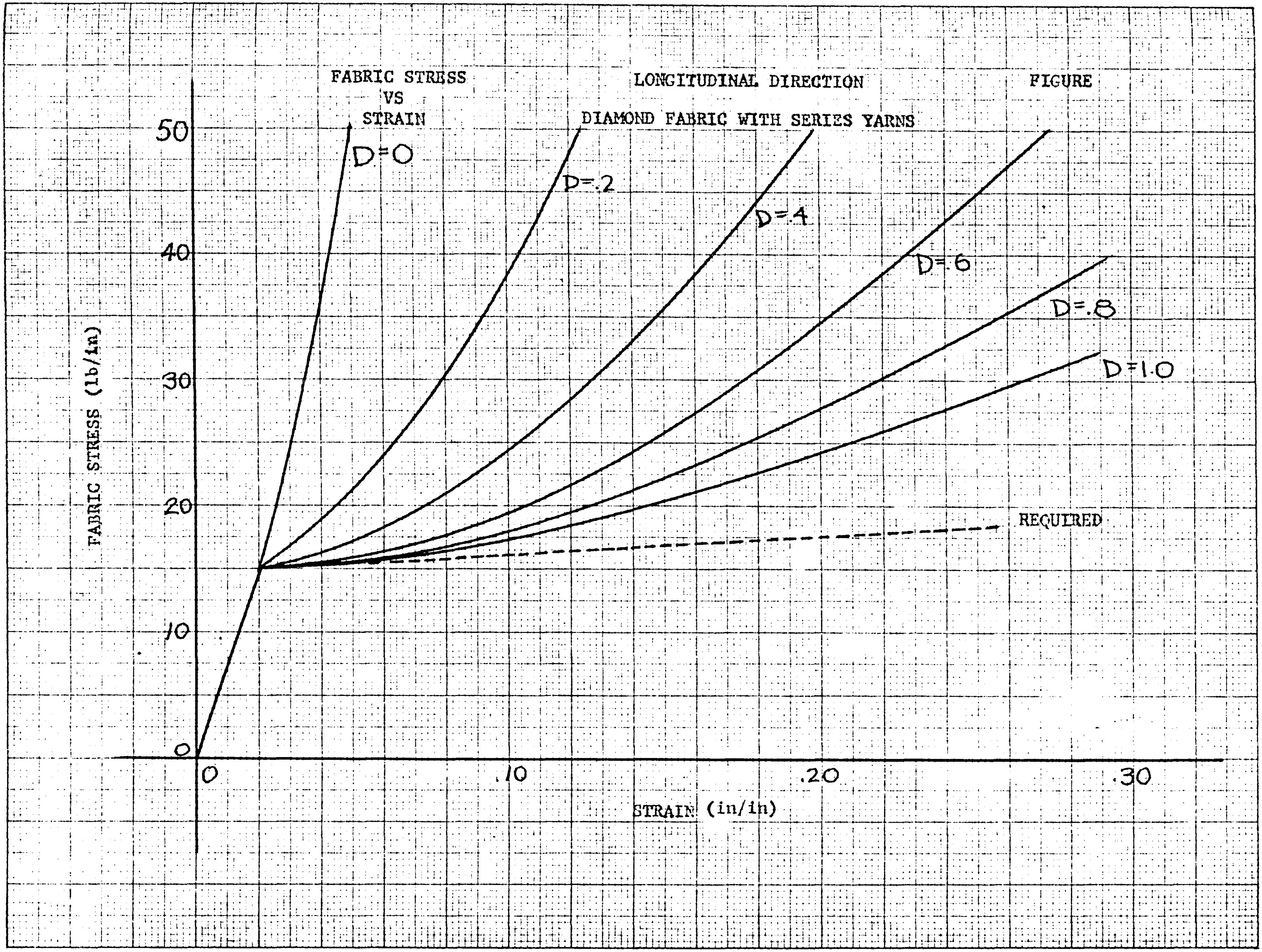
D = fractional length of the diamond elements

Taking the fractional length of the series yarns into account, we can get the shear stress-strain relation for the diamond fabric with series yarns.

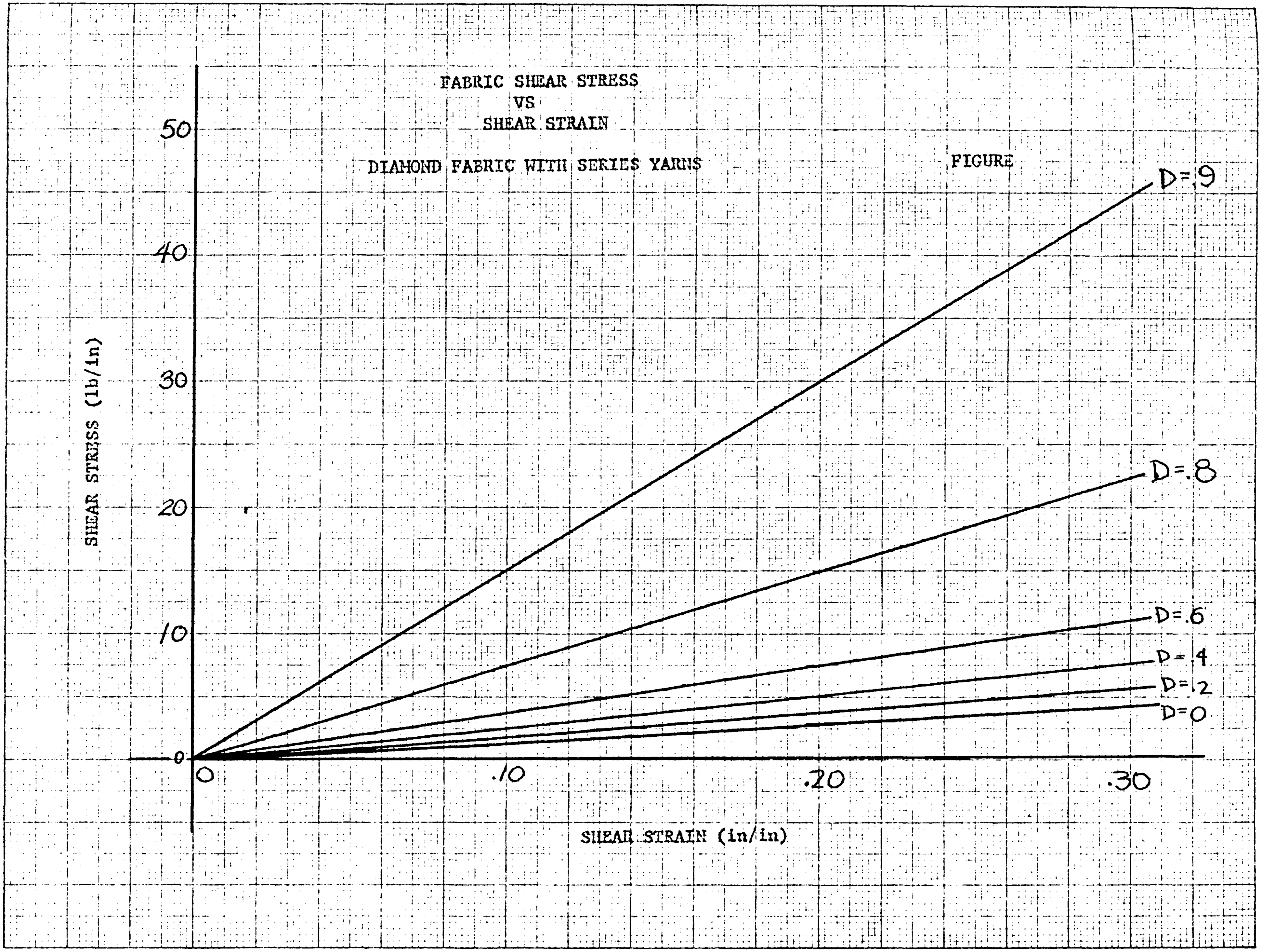
$$\varepsilon_s = (1-D)\frac{S_s}{S_y}$$

$$S_s = \frac{15}{(1-D)}\varepsilon_s$$

This equation is plotted on Figure 49. for various values of D. As expected, in this case, the greater the fractional length of the diamond, the steeper the shear modulus of the fabric.



FABRIC STRESS
VS
STRAIN
LONGITUDINAL DIRECTION
DIAMOND FABRIC WITH SERIES YARNS
FIGURE
D=0
D=.2
D=.4
D=.6
D=.8
D=1.0
REQUIRED
FABRIC STRESS (lb/in)
50
40
30
20
10
0
0
.10
.20
.30
STRAIN (in/in)



FABRIC SHEAR STRESS
VS
SHEAR STRAIN
DIAMOND FABRIC WITH SERIES YARNS
FIGURE
SHEAR STRESS (lb/in)
50
40
30
20
10
0
0
.10
.20
.30
SHEAR STRAIN (in/in)
D=.9
D=.8
D=.6
D=.4
D=.2
D=0

RESIDUAL STRESS YARN FABRIC

Until now, the method used in obtaining the desired stress-strain properties was to put yarns that were assumed either rigid or Hookian into a fabric structure. The properties of the fabric were then largely due to the nature of the structure.

If we now change the properties of the yarn itself, it may be easier to meet the requirements. The question is, how would we like to change the properties of the yarn, and how can it practically be done?

The ideal stress-strain curve that the yarn could have is one which looks like the desired fabric curve. That is, the yarn should be rigid at first, and then easily extensible. This type of yarn* can be produced by taking rubber filaments, stressing them, and then wrapping a yarn around them tightly. When the stress in the rubber is released, the rubber will not go back to its original length because the "wrapped around" yarn will prevent it from doing so. This yarn will hold a residual tensile stress in the rubber.

The stress-strain system of this new combination yarn will be as desired. As stress is applied it will go toward relieving the residual compressive stress in the wrapping yarn before it can cause any elongation. Then when all the residual stress is removed, the combination yarn will extend with its original modulus.

A good approximation to the actual stress-strain curve of this type of yarn is a straight line which intersects the stress axis at

---

*Proposed by Professor Stanley Backer, M.I.T.

a value of stress equal to the residual stress in the rubber, and a slope equal to the modulus of the rubber.

Plain Weave with Residual Stress Yarns

The next step in designing a fabric with these yarns is to determine analytically their required properties. Consider first a loosely woven configuration with the residual stress yarns in line with what will be the longitudinal direction of the fabric tube. If we let:

S_y	=	fabric stress (lbs/in)
F	=	force in yarn (lbs)
F_R	=	residual compressive force in the rubber (lbs)
E	=	modulus of the rubber (lbs/in^2)
P_1	=	spacing between longitudinal yarns (in)
A	=	area of rubber (in^2)
$\mathcal{E}_y$	=	fabric strain (in/in)
$\mathcal{E}_{yn}$	=	yarn strain (in/in)

$$S_y = \frac{F}{P_1}$$

$$\mathcal{E}_y = \mathcal{E}_{yn}$$

Up to the pressure point:

$$0 < F < F_R$$

$$0 < S_y < \frac{F_R}{P_1}$$

$$\mathcal{E}_y = 0$$

After pressure:

$$F > F_R$$

$$S_y > \frac{F_R}{P_1}$$

$$F = EA\mathcal{E}_y + F_R$$

$$S_y = \frac{EA}{P_1}\mathcal{E}_y + \frac{F_R}{P_1}$$

This is the fabric stress-strain equation. Knowing this, we can now determine the required yarn properties. For the fabric to resist the pressure stress without elongation:

$$\frac{F_R}{P_1} = 15 \text{ lbs/in}$$

In order to have the required fabric modulus after pressure:

$$\frac{EA}{P_1} = 15 \text{ lbs/in}$$

The total strain imposed on the rubber (zero at zero residual stress) at a fabric strain of 25% is:

$$\mathcal{E}_T = \frac{F_R}{AE} + .25$$

$$= 125\%$$

These requirements may be difficult to achieve. If, for example, the spacing is 0.1 in., the residual force must be 1.5 lbs and EA must be 1.5 lbs (the same by coincidence). Lowering the size of spacing will give the advantage of lowering the residual stress required; but, the modulus will have to be lowered too and that may be a disadvantage. In any event, there will be a limit to the spacing that is dependent on the yarn diameter.

The total extension imposed on the rubber does not appear to present any difficulty since it is possible to produce rubber that is Hookian above 125%.

A fabric of this nature, assuming that the necessary yarns can be made, would have several of the drawbacks that the other fabrics have. It would not be perfect in a suit without pressure because to achieve any deflection, the residual stress would have to be removed by the wearer, That would require a large moment. (Buckling of the yarns under compression would certainly reduce the moment however.) Also, as in the other fabrics, the shear flexibility can never be made to meet the specifications, regardless of the looseness of the weave. Again, joints of some sort must be used. This was shown in the analysis of the series yarn fabric.

<u>Diamond Fabric With Residual Stress Yarns</u>

In a second attempt to use the residual stress yarns, I have considered what happens when they are put into a diamond fabric. For best result, the yarns should reach their critical point at pressurization. Then, any additional stress would cause the diamond elements to extend for two reasons: 1. movement due to change of angle; 2. extension of the yarns. Because of the relatively high tensions that are built up in the diamond element, the yarns will extend easier than in the plain weave. However, the fabric strain is only a fraction of the yarn strain.

The complete derivation of the results shown here will not appear in the thesis because a somewhat similar calculation has been performed in the section on diamond elements with elastic yarns.

The method used in this solution was to employ the fabric stress-strain equation of elastic yarns considering an infinite modulus up to pressurization, and a modulus of E afterwards. By setting the stress to the required amount at 25% strain, and at pressurization, we find that in this fabric:

$$\frac{F_R}{P_1} = 12.9 \text{ lbs/in}$$

$$\frac{EA}{P_1} = 112 \text{ lbs/in}$$

By considering the total deformation that the rubber undergoes, we can find the total extension to be about 36% at a fabric strain of 25%.

This fabric is somewhat better than the plain weave. The yarns need a lower residual force and the total extension of the rubber is lower. A considerably higher modulus can be used, but whether or not this is an advantage remains to be seen. The big advantage of this fabric is that it will be relatively free to move without pressure. Then, strain will take place by angle changes in the diamond elements. The shear problem can be solved partly by using fabric intersections that will slip, or solved completely by using joints.

It is interesting to note that there is a possibility that a residual stress yarn diamond fabric can be made by interknitting rubber yarns under tension in a tricot fabric. The regular yarns in the structure may be able to resist enough compression to hold the required residual stress in the rubber. (See the tricot fabric on page 82).

ANALYSIS OF LOOP MODEL FABRIC

Introduction

In attempting to find an element of structure which has the desired shape stress-strain curve, the phenomenon of buckling was considered. In buckling, rigid behavior is followed by extreme extensibility. An example of this is the behavior of a slender rod under axial compression. There, the initial application of force will cause little deformation; but, when a critical load is passed, the rod will buckle, and large deformations will take place with small additions of load. The question is, how can buckling be made to affect tensile behavior.

The fabric on page 33. makes use of the buckling of a yarn in compression to improve the behavior of the diamond fabric.

Another way in which buckling can be made to give the desired behavior in tension is by means of the loop element.

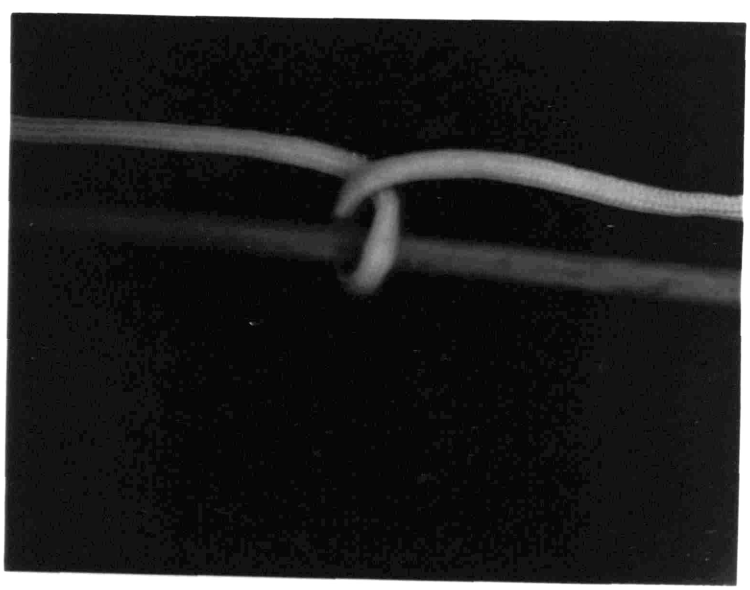


FIGURE 56 LOOP ELEMENT

The loop element consists of a straight rubber filament with a yarn looped around it as shown. When this element is extended, it will have a load elongation curve which is very steep at first, and then after a critical point the load will drop off and build up slowly. This was expêrimentally shown by a qualitative test run on the Instron machine. The fully extended element will have the rubber looped around the yarn, since an exchange of twist takes place in buckling.

Principle of Torsional Buckling in the Loop Element"

When a tension is applied to the element, the yarn exerts a moment on the rubber which tends to twist the rubber into a loop and straighten the yarn. The magnitude of this moment is the yarn tension times the diameter of the yarn. As the force increases, the deflection of the rubber increases, but the moment still stays at the same level. (See Figure 58/a,b) But, at a critical point (Figure 58/c) any further increase of deflection will cause the "moment arms" of the couple to separate and the moment to be equal to a force times a larger distance. (Figure 58/d) If the force is increased beyond the critical point, it will cause the moment to increase. This will cause the deflection to increase, which itself will increase the moment. A vicious circle resulyts, and rapid extension takes place with no increase of force.

Analysis of Loop Element

The purpose of this analysis is to obtain approximate equations for the point on the load elongation curve at which buckling occurs and

A. B. C. D.

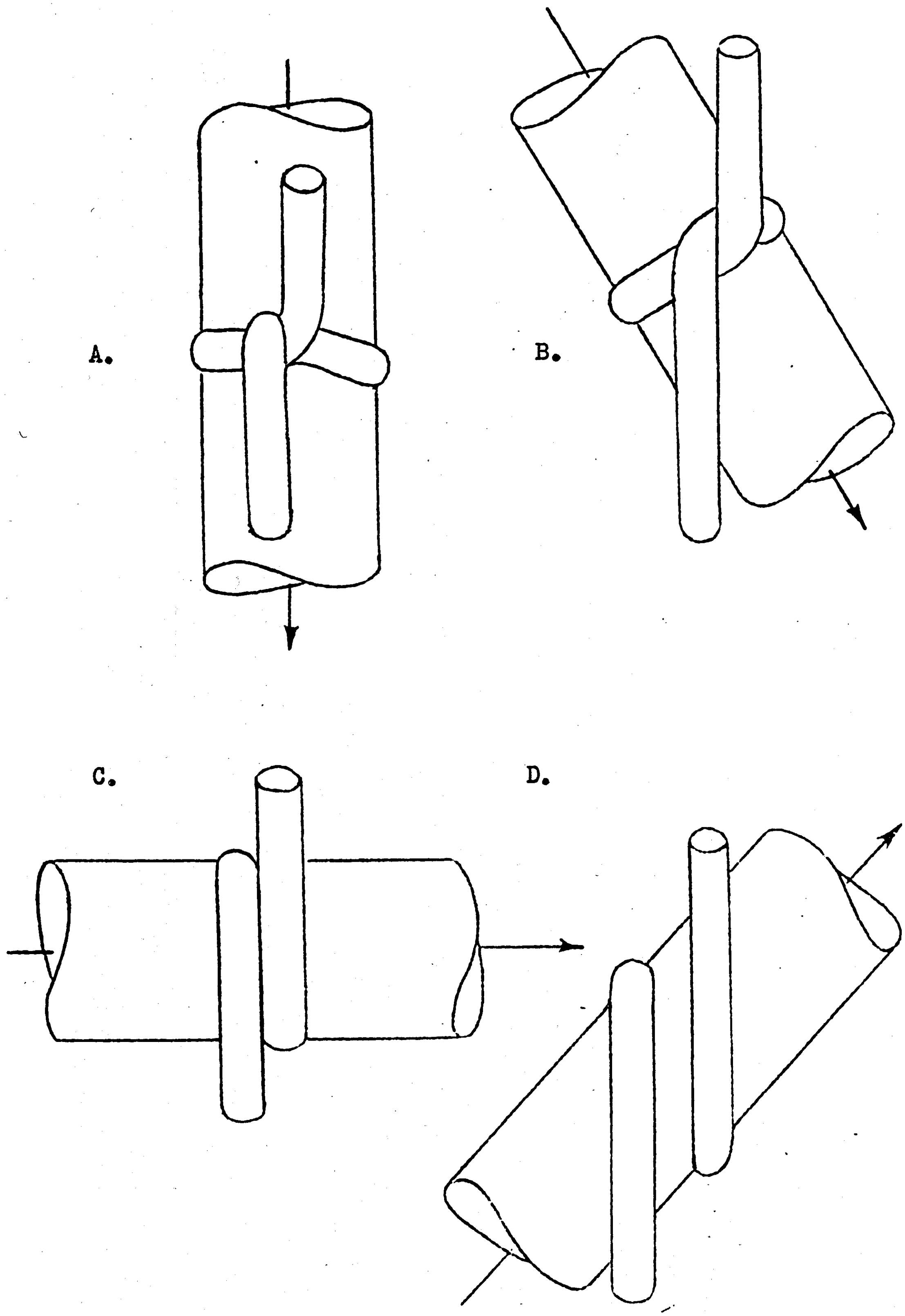


FIGURE 58.

the point at which total extension occurs. Knowing these two points as functions of geometry and properties, it will be possible to analyze a fabric made of loop elements and determine how it can be made to meet the stress-strain requirements.

In order to make this analysis, certain assumptions and approximations had to be made. They are:

1. At the point where the yarn crosses itself, there is enough friction to prevent any slippage.
2. Neither the rubber or the yarn deforms laterally.
3. The energy of bending the rubber is much smaller than its tensile strain energy.
4. The rubber obeys Hooke's Law
5. The yarn is inextensible.
6. The yarn and rubber cross sections are round.
7. The initial geometry is a straight rubber filament with a yarn helix around it.
8. The geometry at any deformation up to buckling will be shown as in Figure 59. . (The yarn cross-section moves around in a circle, where θ is the angle of rotation.)
9. The point of buckling is when an increase in moment will produce an increase of moment arm - when θ is 90° .

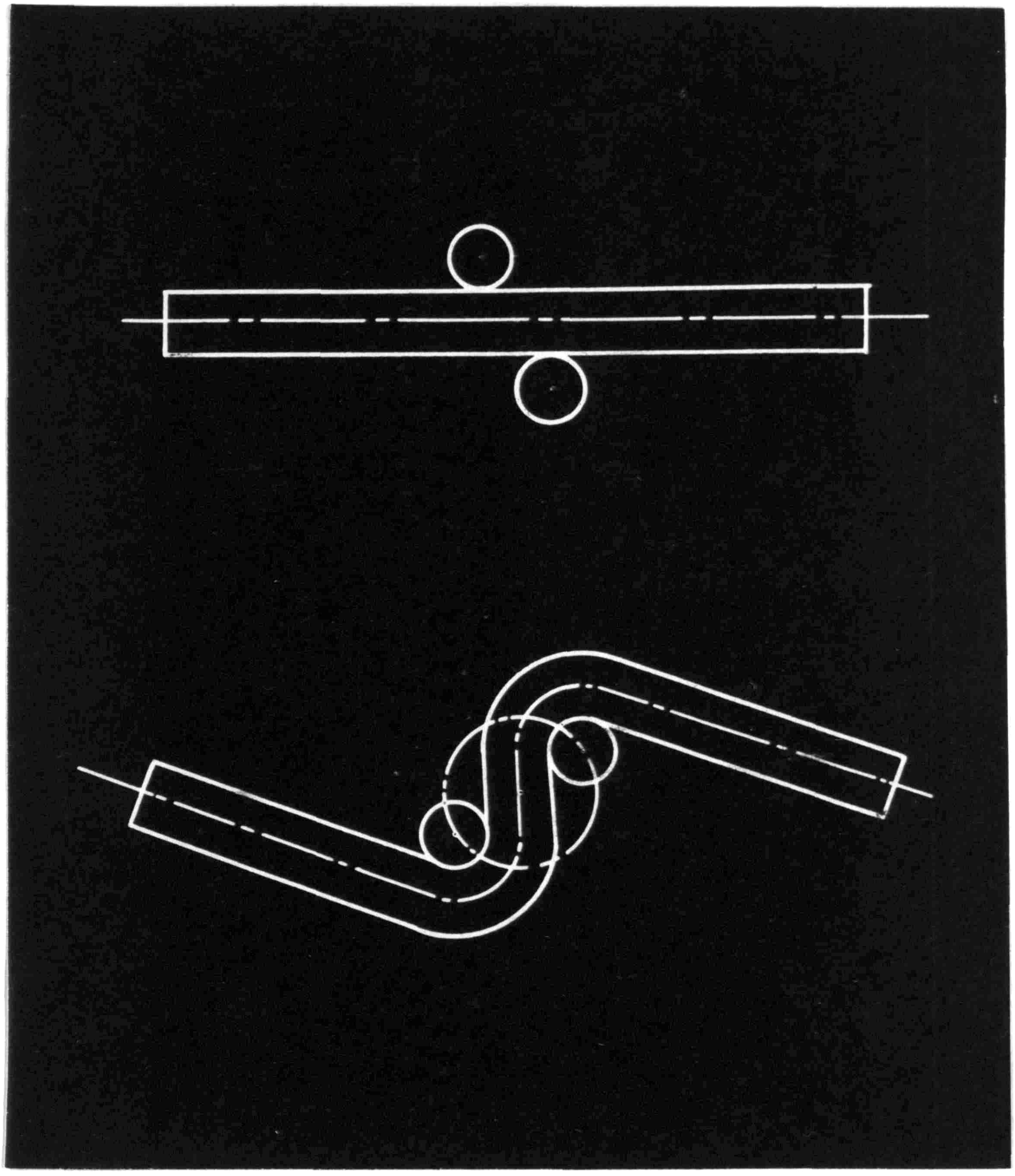


Fig. 60.

The first step is to find the total force at buckling. As explained, this will be when θ is 90°. The total force is the sum of the forces in the yarn and in the rubber.

$$T = F + P$$

Where

P = force in the rubber (lb)

F = force in the yarn (lb)

Since all the strain energy in the rubber is assumed to be due to tensile strain:

$$P = \frac{E \delta \pi R^2}{L}$$

Where:

R = radius of the rubber (in)

E = tensile modulus of the rubber (lb/in^2)

L = original length of element (in)

δ = elongation of rubber (in)

It should be noted that the parallel force component of the rubber is approximately the same as the tension. This is true since the length of the element is large compared to the radius. The above equations make use of this fact.

From the assumed geometry, we can find from Figure 60. that when buckling occurs:

$$\delta = R\left(\sqrt{n^2 - 4n + 52} + 8.28 - n\right)$$

Where $\qquad n = \frac{L}{R}$

R = radius of rubber (in)

R = radius of yarn (in)

Substituting we get:

$$P = \frac{E\pi R^2}{n}\left(\sqrt{n^2 - 4n + 52} + 8.28 - n\right)$$

The force in the yarn is related to the moment which is exerted on the rubber.

$$F = \frac{M}{2R}$$

Where:

M = moment exerted by the yarn (in-lb)

Considering the rubber alone as an isolated body, we can find from the conditions of equilibrium that:

$$M = P n R \frac{6}{(n-4)}$$

By making the necessary substitutions, we can solve for the total force at buckling.

$$T = \frac{P n (3)}{n-4} + P$$

$$T = 4\left(\frac{n-1}{n-4}\right)\frac{E\pi R^2}{n}\left(\sqrt{n^2 - 4n + 52} + 8.28 - n\right)$$

It is not obvious that this force is the maximum force that is encountered up to buckling. However, it was found that the relation between T and θ is monotonically increasing up to ninety degrees. Thus, the T which was calculated in the maximum.

The strain of the element at buckling can be found by considering the angular movement of the yarn at the crossing point and converting it into linear displacement. Using simple trigonometric relations, this was found to be:

$$\mathcal{E} = \frac{2}{n}$$

Having found the point on the load-strain curve at buckling, the problem is now to find the point at total extension of the element. At that time, the yarn will be straight and have approximately zero tension. The force in the system will therefore be due to the rubber tension only; and this can be found from the extension as before.

$$T_T = P_T = \frac{ETR^2}{L}\,\delta_T$$

Where:

T_T = total force at total extension (lb)

P_T = rubber tension at total extension (lb)

δ_T = total rubber extension (in)

Considering the length of the rubber in the totally extended condition (when it is looped around the straight yarn) we get:

$$\delta_T = 29.3R$$

To get the total strain at this point, we consider the difference in length between the straight yarn and the yarn as it is looped around the rubber, and the result is:

$$\mathcal{E}_T = \frac{14.8}{12}$$

Where

$\mathcal{E}_T$ = strain at total extension (in/in)

By substituting arbitrary values for the constants of the system, we find that the force in the element drops off after buckling. This was just as was determined by the qualitative tests.

It would be possible to strain the element somewhat past the calculated point of total extension, but then the load would build up as dictated by the tensile properties of the yarn.

Loop Element Fabric

One way in which the loop element can be used in a fabric is shown on Figure 84 . In that case it would be possible to meet the bending specifications for stress if:

$$\frac{T_B}{P_1} = 15$$

Where:

T_B = force at buckling (derived in Eq. #1)

P_1 = element spacing

The force at total extension does not enter into this requirement since it is lower than the buckling force.

By designing the fabric in this manner, the suit will just be at the buckling point at pressurization. Any additional stress will cause buckling, and the required strain for buckling.

In order to meet the strain requirements, it will probably be necessary to insert portions of the fabric which do not extend. Thus, the fabric strain will be a fraction of the loop element strain.

$$\varepsilon_F = B\varepsilon_E$$

Where

$B =$ fractional length of elements

$\varepsilon_F =$ fabric strain

$\varepsilon_E =$ element strain

In order to fill the pressure strain requirement:

$$\frac{B\,2}{n} \leq .02$$

In order to meet the bending strain requirement:

$$\frac{B\,14.8}{n} \geq .25$$

These last equations lead to a slight inconsistency which was brought about by the approximations that were made. Nevertheless, it can be shown that the required relation between n and B is:

$$\frac{n}{B} = 100$$

Shear Stress-Strain Relation

The shear flexibility of the loop fabric will be almost the same as that of a fabric with only straight yarns in the longitudinal direction. The stress-strain curve will be similar to that of the Diamond Fabric with Series Yarns, where D is equal to zero.

BENDING AND TORSIONAL RIGIDITY OF TUBES WITH DIFFERENT RADII

Introduction

In the previous sections of this thesis, several fabrics structures were analyzed to determine their stress-strain behavior under biaxial stress. An attempt was made to design these fabrics so that they would meet certain specifications for bending and twisting while under internal pressure. However, these specifications were written only for a tube of three inch radius. Since a complete pressure suit would necessarily contain tubes of many different radii, it is of great importance to determine how the overall behavior of a fabric tube is affected by its size.

The purpose of this analysis is to find how the bending moment varies with curvature and how the twisting moment varies with torsion for tubes of different radii.

The fabric structure which will be considered here is the diamond fabric. That is because this fabric most nearly has the required stress-strain curves.

Bending Moment Analysis

From Strength of Materials we know that for a thin-walled cylinder made of elastic material, the following holds:

$$S_{yb} = \frac{M_b}{\pi R^2}$$

$$M_b = \pi R^2 S_{yb}$$

Where:

S_{yb} = stress due to bending (at outside of bend) (lb/in) (lb/in)

R = radius of cylinder (in)

M_b = bending moment (in-lb)

In the analysis of the diamond fabric, we found that:

$$S_y = \frac{S_x (1+\varepsilon_y)^2}{3-(1+\varepsilon_y)^2}$$

Where:

S_y = total longitudinal stress at strain (lb/in)

S_x = stress in circumferential direction (lb/in)

This stress S_y is the stress due to both bending and pressure. To get the bending stress, we must subtract from this the pressure stress in the longitudinal direction. Thus:

$$S_{yb} = S_y - S_{yp}$$

S_{yb} = stress due to bending alone (lb/in)

S_{yp} = stress due to pressure alone (lb/in)

In bending, there is no circumferential stress, therefore will be due to pressure alone. From the stress relations of a* torus under internal pressure, we can approximate the stresses at the outside of a bent cylinder to be:

$$S_{yp} = \frac{PR}{2}$$

$$S_x = P \cdot R \frac{1}{2} \left[\frac{2+RK}{1+RK} \right]$$

* ref. 1

Where:

P_o = Pressure (lb/in^2)

K_o = Curvature (in^{-1})

It is important to note that the stresses which we are concerned with here are those that occur at the outside of the bend, - at the most stressed portion of the cylinder.

The moment equation holds only if the bending stress varies linearly with the distance from the center of the tube from zero to in tension and zero to S_{yb} in compression. However, as can be seen from the stress-strain equation of the diamond fabric, neither of these requirements is fulfilled. The stress does not vary linearly, and it varies in a different way in compression. Nevertheless, the moment equation will be used in this analysis to given an approximate solution, and I will state in which way the error will occur. It would be impossible to obtain an exact solution to this problem in any event since the geometry of deformation is not known.

Effect of Using Approximate Solution

The approximate solution yields values for moment which will be considerably higher than the actual results. Several reasons for this will now be explained.

The relation between bending stress and strain, or bending stress and distance from the neutral axis is on which is almost linear, but which has an increasing derivative. Since the expression for moment is:

$$M = \int_A y\sigma \, dA = \int_S S y \, ds$$

Where:

y = distance from neutral axis

S = circumferential length

an approximation which considers S_{yb} to vary linearly will give a moment which is above the actual value. This can easily be seen by making a sketch of any increasing derivative function and comparing the value of its integral with the integral of a straight line from zero to the end-point.

The second way in which this fabric violates the conditions of the original moment equation is that the behavior in compression is not the same as in tension. (Note that by compression, I mean the relieving of tensile stress due to pressure.) The compressional stress relation with distance from the neutral axis is at every point lower than the tensile stress relation in the other direction. This means that putting the bottom of the tube into compression will be "easier" than putting the top into tension. Thus, for a second reason, the moment found by the approximation will be higher than actual.

Another error is introduced because of the difference in stress distributions of tension and compression. From equilibrium requirements, it can be shown that the areas of both distributions must be equal. To do this, the neutral axis will have to shift towards the outside of the bend. That will cause the stress at the outside to be less than calculated and will cause the theoretical moment to be considerably less than the actual one.

There is one other important factor which will cause this analysis to be in error and that is the fact that the tube will take on an elliptical cross-section to lower its total strain energy*, and consequently the stress. This will also cause the calculated moment to be high.

With all these reasons for error, it is doubtful if this analysis will predict actual behavior very accurately. However, it will certainly give a qualitative picture of how moment, curvature, and tube radius are related.

Continuing with the approximate analysis we substitute into the moment equation and get:

$$M = \pi R^2 \left[S_y - S_{yp} \right]$$

$$= \pi R^2 \left[\frac{S_x (1+\mathcal{E}_y)^2}{3-(1+\mathcal{E}_y)^2} - S_{yp} \right]$$

$$= \frac{\pi P R^3}{2} \left[\frac{(1+\mathcal{E}_y)^2}{3-(1+\mathcal{E}_y)^2} \left(\frac{2+RK}{1+RK} \right) - 1 \right]$$

Since the strain in this equation is the strain at the outside of the bend:

$$\mathcal{E}_y = RK$$

$$M = \frac{\pi P R^3}{2} \left[\frac{(1+RK)(2+RK)}{3-(1+RK)^2} - 1 \right]$$

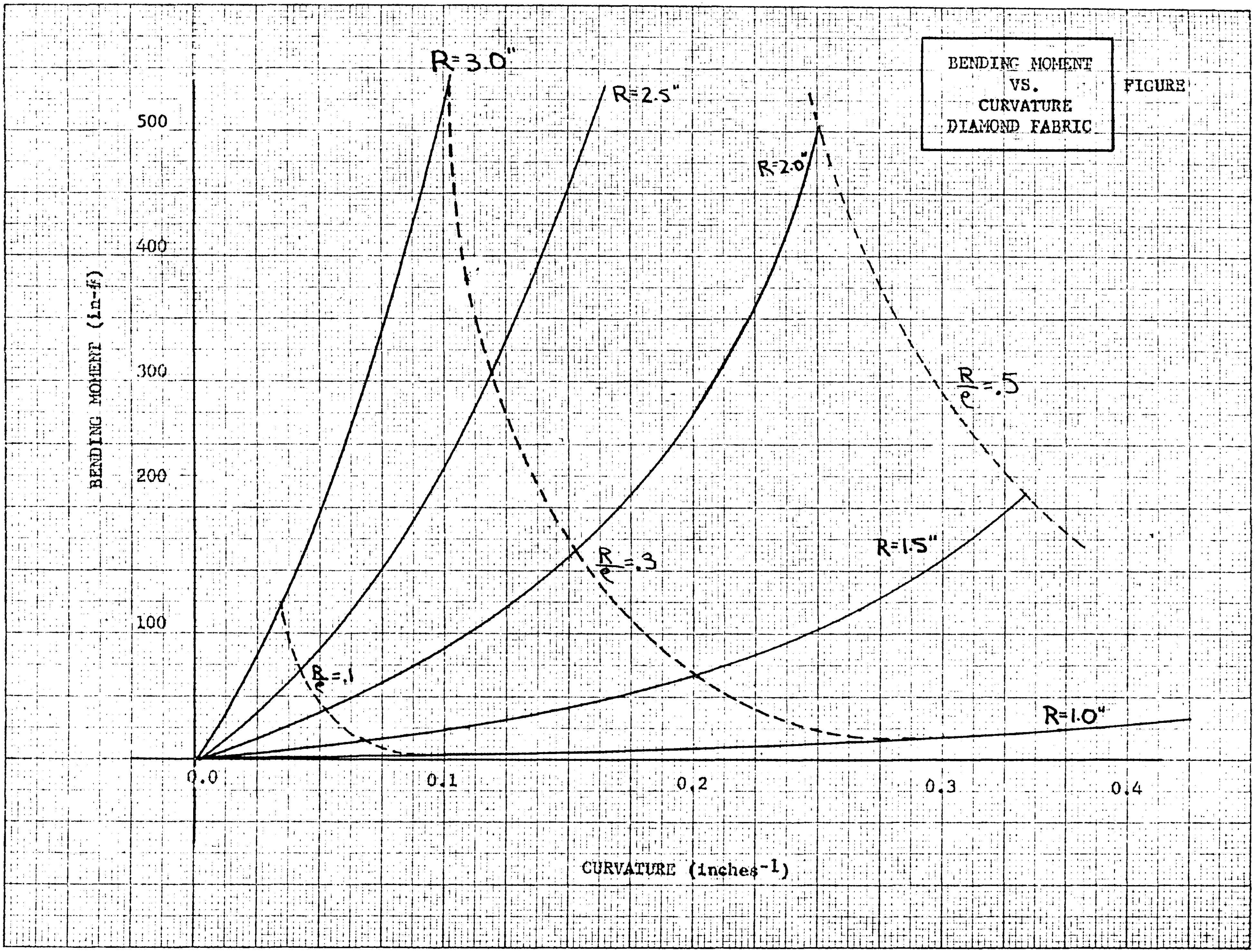
This is the desired equation. It is plotted on Figure 72. and values appear in Table I . Lines of equal R/radius-of-curvature are indicated to show the "amounts of bending" that takes place in the tubes at various curvatures.

* ref. 2.

TABLE I.

Bending Moment vs Curvature for Diamond Fabric Tubes Under 10 psi Pressure

	R=1"		R=2"		R=3"	
$\frac{R}{\rho}$	K	M_B	K	M_B	K	M_B
0	0	0	0	0	0	0
.1	.1	4.55	.05	36.4	.033	123
.2	.2	10.9	.10	87.7	.067	294
.3	.3	20.1	.15	161	.100	543
.4	.4	35.0	.20	280	.133	945
.5	.5	62.8	.25	503	.167	1700


BENDING MOMENT
VS.
CURVATURE
DIAMOND FABRIC
FIGURE
BENDING MOMENT (in-#)
500
400
300
200
100
0.0
0.1
0.2
0.3
0.4
CURVATURE (inches-1)
R=3.0"
R=2.5"
R=2.0"
R=1.5"
R=1.0"
R/e=.1
R/e=.3
R/e=.5

Twisting Moment Analysis

The twisting moment of a thin-walled cylinder is:

$$M_T = 2\pi R^2 S_s$$

Where:

M_T = Twisting moment (in-lb)

S_s = Shear stress at cylinder surface (lb/in)

R = Radius of cylinder (in)

In the analysis of the diamond fabric, it was found that the best freedom to shear was obtained by yarns slipping at the intersections. If the static and dynamic coefficients of friction are low (about .01) then the shear stress-strain relation can be written (with about 90% accuracy) as:

$$S_s = S_y \mathcal{E}_s$$

Where:

$\mathcal{E}_s$ = shear strain (in/in)

S_y = total stress in longitudinal direction (lb/in)

τ = angular deflection per length (radians/in)

$$\mathcal{E}_s = R\tau$$

If the twisting takes place at a time when there is no bending, then all the longitudinal stress will be due to pressure alone, and:

$$S_y = \frac{PR}{2}$$

Substituting:

$$M_T = 2\pi R^2 \frac{PR}{2} R\tau$$

$$M_T = \pi P R^4 \tau$$

This is the desired relation. It is shown plotted on Figure 75. and tabulated in Table I for several values of R and τ,

TABLE II.

Twisting Moment vs Torsion for Diamond Fabric (with slipping intersections) Under 10 psi

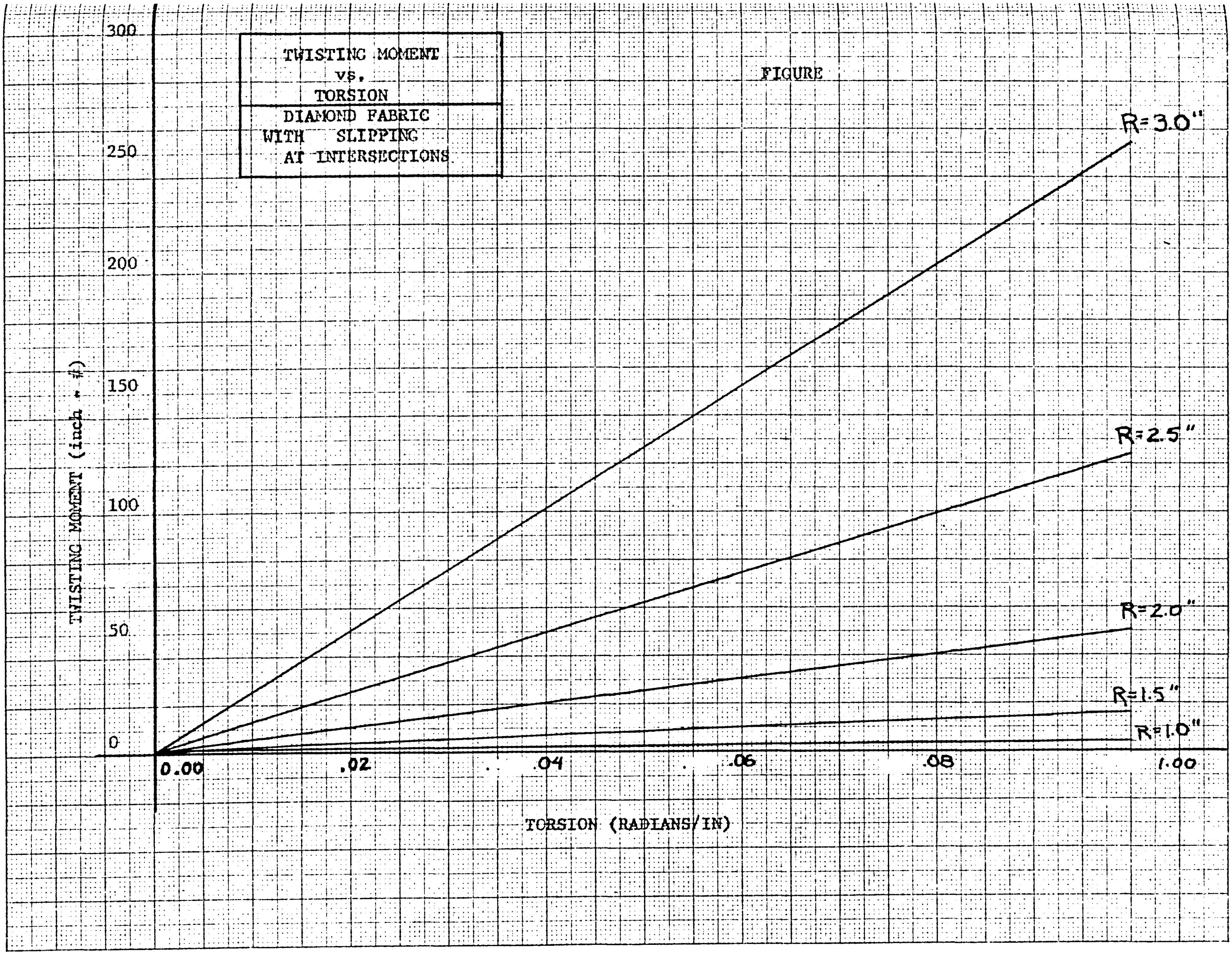
τ (rad/in)	R = 1" M_T (lbs in)	R = 2" M_T	R = 3" M_T
0	0	0	0
.02	0.5	10	51
.04	1.2	20	102
.06	1.8	30	153
.08	2.4	40	203
.10	3.0	50	254

Discussion

In this analysis approximate equations were derived for bending and twisting moments as a function of curvature and torsion. These relations are close to linear, and, by definition, their slopes are the bending and torsional rigidities.

As the radius of the tube is increased, both the bending and torsional rigidity increase greatly. They increase with the fourth power of radius. This means that in order to obtain the lowest possible resistance to bending and twisting, the radius of the tubes should be as small as possible. The suit should be made to fit tightly.

There is a possibility that an additional stress will build up due to fabric jamming against the body in the circumferential direction in bending if the fit is too tight. However, since the bottom of the tube expands in bending, the net contraction will be very small.


TWISTING MOMENT
vs.
TORSION
DIAMOND FABRIC
WITH SLIPPING
AT INTERSECTIONS
FIGURE
R=3.0"
R=2.5"
R=2.0"
R=1.5"
R=1.0"
300
250
200
150
100
50
0
TWISTING MOMENT (inch - #)
0.00
.02
.04
.06
.08
1.00
TORSION (RADIANS/IN)

Comparing the results obtained here to the requirements, we find that neither the bending rigidity, nor the torsional rigidity of the three inch tube is as low as it should be. The results will be discussed in detail in the General Discussion.

MANUFACTURE OF FABRIC STRUCTURES

The object of this section is to present methods of making the fabric structures which were previously analyzed.

It is important to note that a straight line in the fabric structure diamgram can represent either a straight yarn or any other arrangement of yarns which will form a straight line. For example, the lines of the diamond element can be made by chains of yarn, as in the tricot fabric.

The first set of fabrics are all diamond fabrics. Several have special advantages and they will be discussed in detail.

DIAMOND FABRICS

Any fabric which has structural elements that are arranged in a diamond array is considered a diamond fabric. The following fabrics do not include every possible textile fabric of this type, but they do include a wide variety that are made by many different methods of fabrication.

Fabric #1 - Knotted Fabric

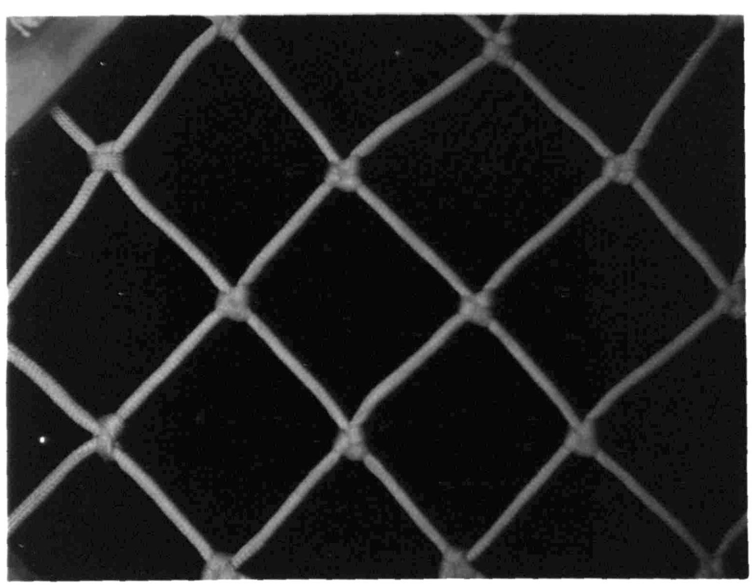


FIGURE 77. KNOTTED FABRIC

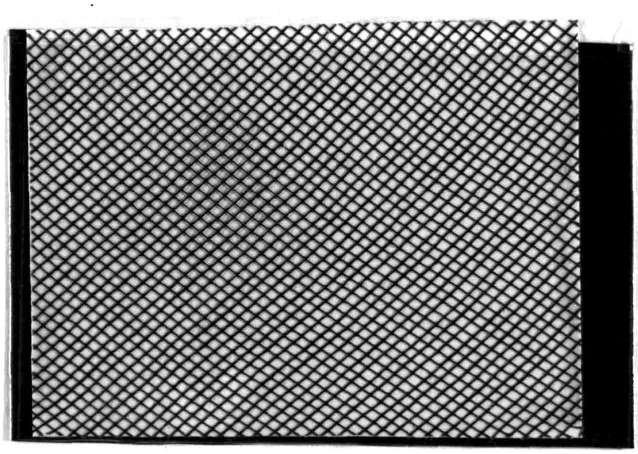
This fabric is made on a knotting machine. The knots are weavers knots (can be made otherwise) and they do not permit any slippage at all in the intersections. This fabric will not be affected by the breaking of a single yarn. Because of the knotted intersections, it can be very easily used with wire joints. (See page 85).

Fabric #2 - Braided Fabric

This fabric can be made in tubular form. However, its diameter is limited to a few inches by practical considerations. The intersections will permit a form of slippage; but, they will not allow shear deformation to take place easily. (In order to have slippage increase the shear flexibility, it must take place similar to the way it does in the bobbinet fabric.)

The structure of the fabric is exactly the same as the distorted plain weave.

Fabric #3 - Distorted Plain Weave

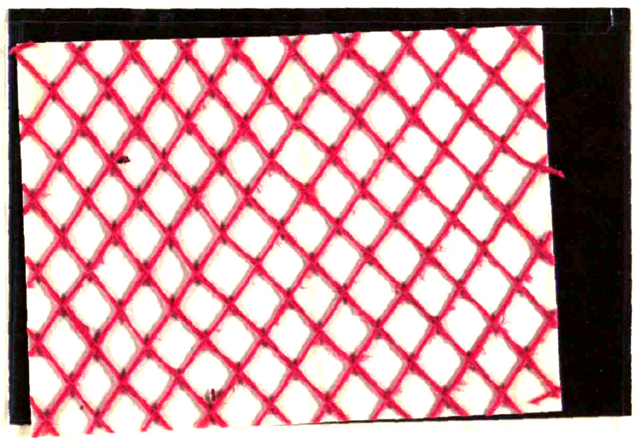


SAMPLE: 78. DISTORTED PLAIN WEAVE (ALSO BRAIDED FABRIC STRUCTURE)

This is a plain weave fabric which has been woven with the same number of ends as picks per inch. Then it has been stretched to the desired angle and set.

It can be made as wide as desired, but not in tubular form. The slippage situation is exactly the same as in the braided fabric.

Fabric #4 - Distorted Leno Weave



SAMPLE:79. DISTORTED LENO WEAVE

This fabric is made like the distorted plain weave, except for the fact that a leno fabric is used. The advantage of this fabric over the distorted plain weave is that it can be made with relatively large intersections and still remain dimensionally stable. It will not permit slippage in the intersections.

Due to the fact that this fabric is not symmetrical, it may have different properties as it is twisted in opposite directions. This may be of some advantage.

Fabric #5 - Bobbinet Fabric

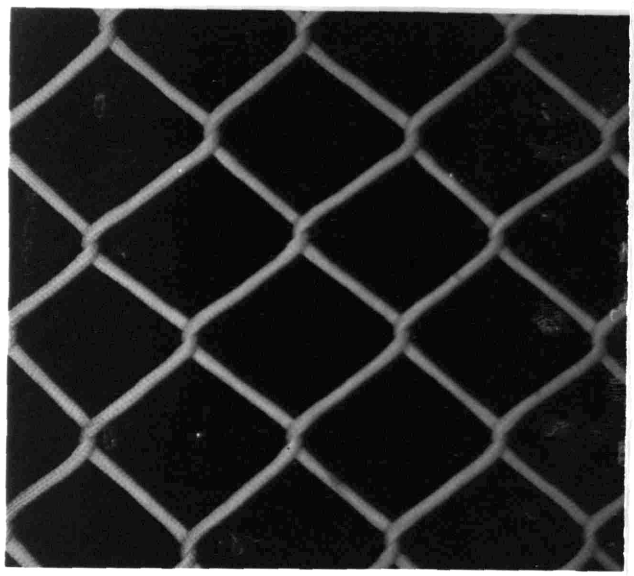


FIGURE 80/a BOBBINET FABRIC

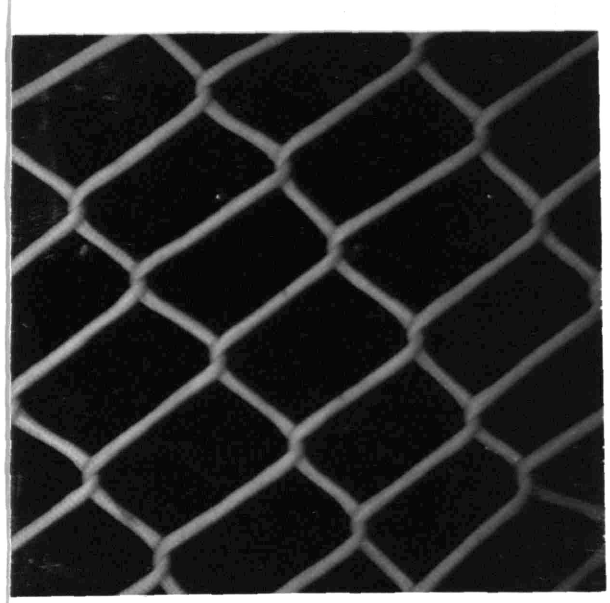


FIGURE 80/b BOBBINET FABRIC WITH SHEAR DEFORMATION

This fabric* has the outstanding advantage of permitting slippage to occur at the intersections. It can be used either as shown in the photograph, or rotated 90° without a change in twisting properties.

This fabric is made on a bobbinet machine, which is a type of lace machine.

As the fabric is shown, the breaking of a single yarn would cause serious damage to the fabric. This can be avoided if an additional yarn is twisted around the strands in this structure in such a way that it will not influence the strength properties, but will hold the fabric together if a strand breaks. (See figure 81/a).

*Proposed by B. J. Park, Graduate Student, M.I.T.

This fabric can be made with straight yarns in series with the diamond elements.

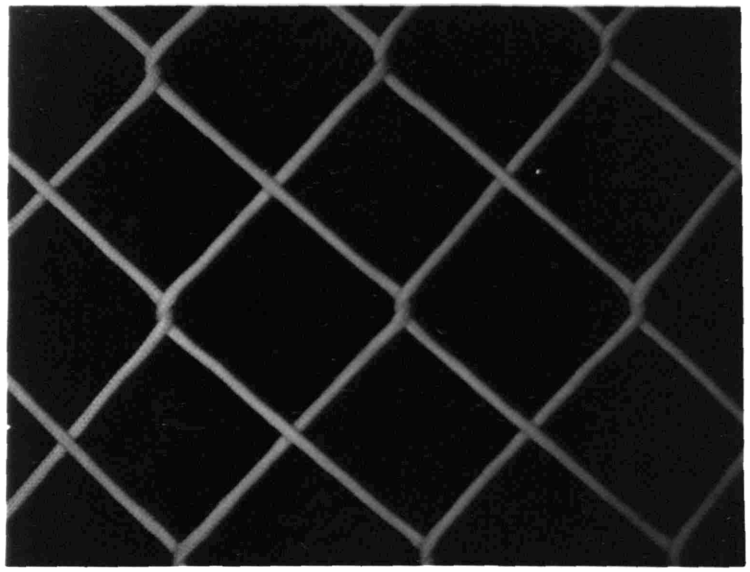


FIGURE 81/a MODIFIED BOBBINET FABRIC

Fabric #6 - Special Leno Fabric

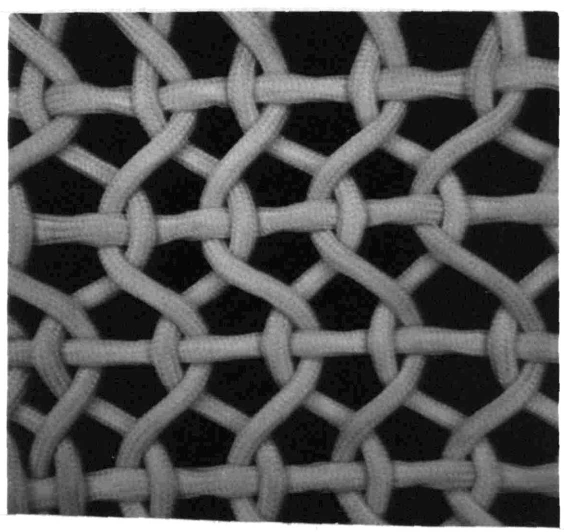
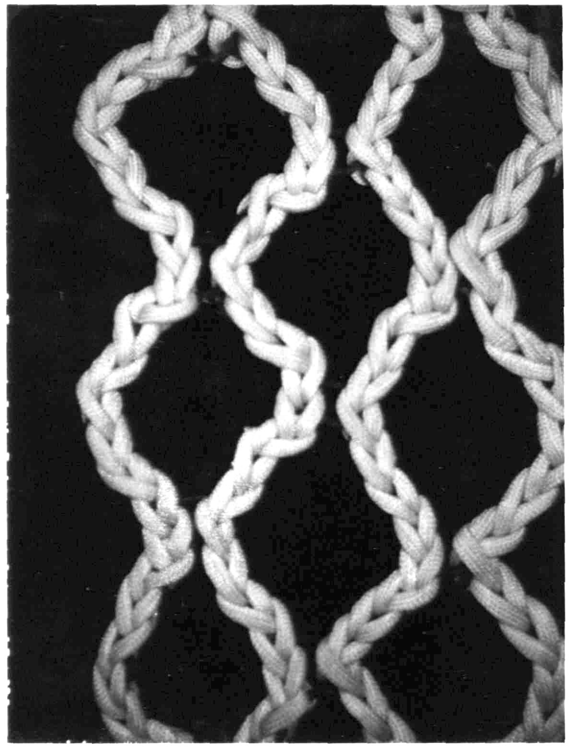
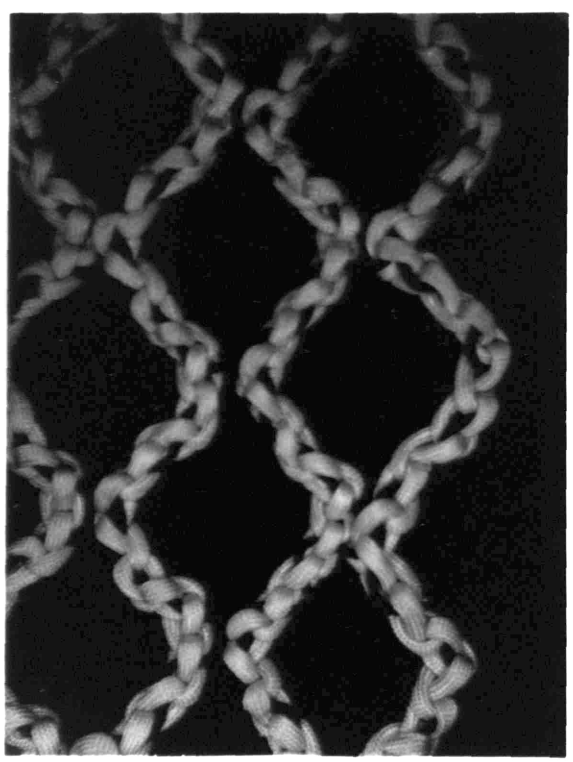


FIGURE 81/b SPECIAL LENO FABRIC





This fabric can be made with yarns in it which will resist compression. (Horizontal yarns on photograph). It will then have the properties which were previously determined.

This fabric can be made in tubular form and it can be made with series yarns.

Fabric #7 - Chain Tricot Fabric

Sample: 82/a

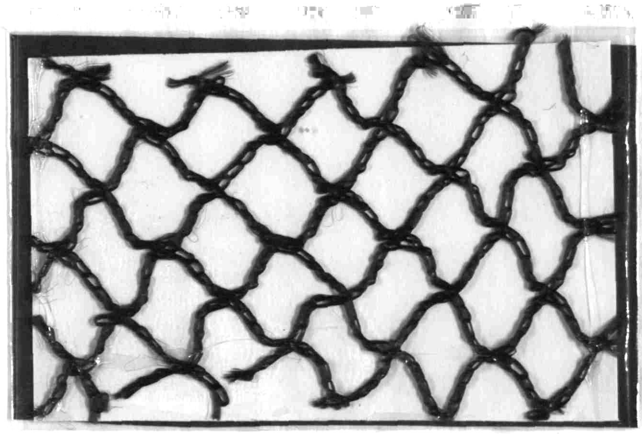


FIGURE 82. CHAIN TRICOT FABRIC

SAMPLE CHAIN TRICOT FABRIC

b. c.

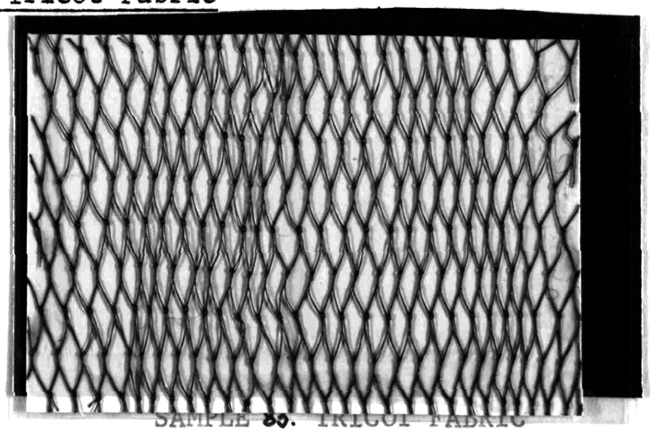
This fabric is made on a tricot knitting machine. It is made by knitting chains of yarn with stuffer yarns inside. At certain intervals, the stuffer yarns of one chain intersect with an adjacent chain. In this way, the stuffer yarns pull the chains together to form the diamond structure.

This fabric can be made with straight yarns in series with the diamond elements. The straight yarns will actually be chains with stuffer yarns. By varying the tension of the stuffer yarns, the elasticity of the series yarns can be controlled.

Some slippage will probably take place at the intersections.

In the event one yarn breaks, the fabric will still retain its structure. This is because there are both chain yarns, and stuffer yarns involved.

Fabric #8 - Tricot Fabric



This fabric is made on a tricot knitting machine; it contains two sets of yarns. One set forms the interlooping and the other set forces the long loops into this particular structure.

By using very smooth yarns, it is possible to obtain slippage in the intersections of this fabric. Straight series yarns can be put in since this fabric is knit on the bias, as shown.

The breaking of a single yarn will not completely ruin this fabric. However, there is a possibility that a run will form if the yarns are very smooth.

LOOP FABRIC

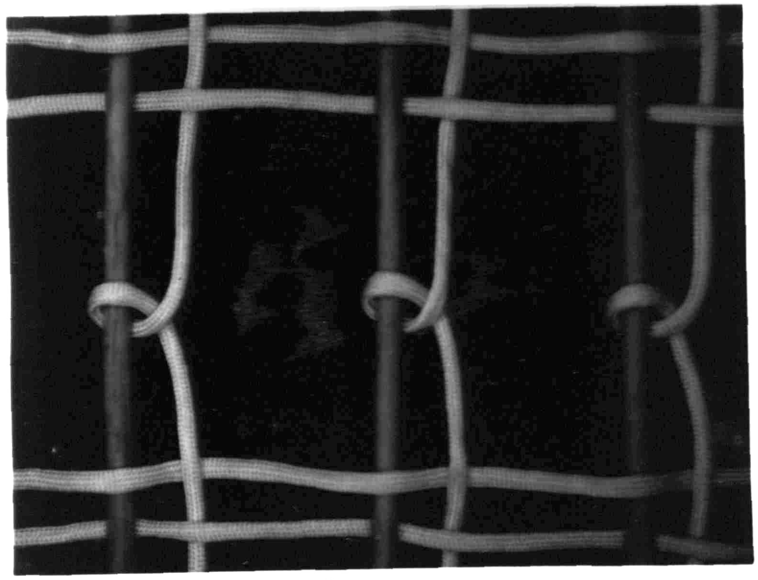


Fig. 84.

This fabric contains a series of loop elements surrounded by plain weave. The yarn that makes the loop must have a supply package that can be easily manipulated in the making of the fabric. It must be passed around another yarn and then over itself. This motion is very complicated and cannot be performed on a loom unless extremely intricate attachments are first designed and made.

The fabric can be made completely by hand, but that would be very time consuming.

Bending Joints

The most practical use for the loop element would be in modifying existing fabrics which do not meet the requirements. They would be useful in places where large tensile strains are imposed by bending. Because of their structure, they would allow the fabric to have large local extensions.

The operation of modifying a fabric in this manner cannot be done by machine and must be done manually. All that is required is to insert rubber filaments into a few critical places.

The parts of the fabric that are to be modified must be made slightly oversize. Then, the yarns which will be most strained are to be made into loops through which rubber filaments are passed. The filaments must be anchored at both their free ends to some convenient part of the fabric. The resulting structure is made up of the original fabric with loops elements instead of straight yarns in certain places. The modified portion of the fabric would effectively be a bending joint.

SHEAR JOINTS

In order to make a pressure suit with sufficient shear flexibility, it may be necessary to include joints in critical places of the fabric. The joints would join two tubes together and permit twisting to occur. They could be made of various types of bearing arrangements. However, the simplest method and probably the best is one which just uses a stiff wire.

The two tubes to be connected are joined to the wire as shown in Figure 86 . The most convenient fabrics to use for this are the bobbinet fabric, and the knotted fabric.

The coefficient of friction between wire and yarns must be small, and the lateral displacement before jamming large. It may be necessary to use several joints at certain parts of the body such as the arms, and the waist.

It is important to note that the wire must be stiff enough to keep from bending as the loops pull it. If it bends, then the situation will be exactly the same as in the plain bobbinet fabric with no joints.

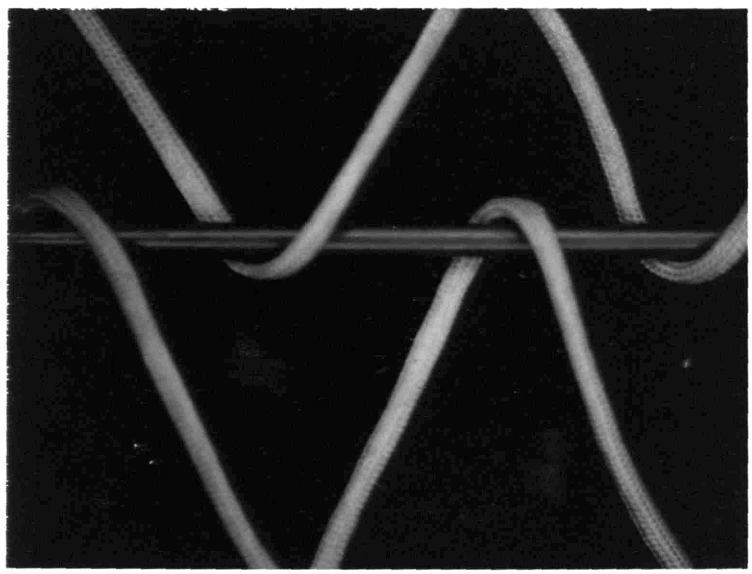


FIGURE 86. WIRE SHEAR JOINT

RESULTS

Redefined Physical Properties

Cylinder Radius	-	3 in.
Internal Pressure	-	10 lb/in.

Pressure Requirements

Maximum allowable strains

Longitudinal	-	.02
Circumferential	-	.06

Bending Requirements

Required Curvature	-	.0785
Allowable Bending Moment		100 in-lb

Twisting Requirements

Required Torsion	-	.0785
Allowable Twisting Moment	-	15 in-lb

Fabric Stress-Strain Requirements

Longitudinal Direction

Pressure Stress	-	15 lb/in
Allowable Strain	-	.02
Required Bending Strain	-	.25
Allowable Stress	-	18.5 lb/in

Circumferential Direction

Pressure Stress	-	30 lb/in
Allowable Strain	-	.06

Shear Stress Strain

Required Strain = .236

Allowable Stress = .215 lb/in

Fabric Stress-Strain Results

The fabrics presented here are designed to meet the pressure requirements. These results are for bending and twisting.*

Fabric	Longitudinal Tensile Stress at the Required Strain (lb/in)	Shear Stress at the Required Strain (lb/in)	Special Requirements
Required	18.5	.215	—
Perfectly Elastic Material	187.5	2000	$E = 750$ lbs/in
Diamond Fabric with Slipping at Intersections	32.7	3.5	$\theta_0 = 35°$ μ small *(p. 20.)
Diamond Fabric with Rigid Intersections	32.7	21.3	$\theta_0 = 35°$
Diamond Fabric with Elastic Yarns	28.8	3.5 or 21.3	$EA/W = 1{,}140$ lbs/in *(p. 38.)
Diamond Fabric with Compression Resistant Yarns	18.5	21.3	$\frac{Q}{P_{20}} = .54$ lbs/in $\theta_0 = 29.7°$ *(p. 35.)
Residual Stress Yarn Diamond Fabric	18.5	3.5 or 21.3	$\frac{F_R}{P_1} = 12.9$ lbs/in $\frac{EA}{P_1} = 112$ lbs/in *(p. 54.)

* See next page

Residual Stress Yarn Plain Weave	18.5	3.5	$\frac{F_R}{P_1} = 15$ lbs/in $\frac{EA}{P_1} = 15$ lbs/in *(p. 52.)
Loop Element Fabric	18.5	3.5	*(p. 64-5.)
Diamond Fabric with Series Yarns	See Figure 48.	See Figure 49.	$\frac{AE}{P_{2_0}} = (1-D)\,1060$ lbs/in *(p. 43.)

Results of a Tube of Diamond Fabric with Slipping at Intersections

Bending

Required Curvature = $.0785^{-1}$

Allowable Moment = 100 in-lb

Theoretical Moment = 400 in-lb

Twisting

Required Torsion = .0785 in^{-1}

Allowable Moment = 15 in-lb

Theoretical Moment = 250 in-lb

Effect of Tube Radius

$$M_b \propto K R^4$$
$$M_T \propto \tau R^4$$

*The explanation of the symbols used will be found on the pages indicated in parenthesis).

DISCUSSION AND CONCLUSIONS

Introduction

This section contains a discussion of the results of the various calculations performed in this thesis. On the basis of the results, it was possible to reach a conclusion as to which fabric is most likely to satisfy the requirements for a pressurized suit. The results will be discussed in the order which they appear in the previous sections.

Redefined Physical Properties

From the specifications that were given, it was possible to determine the required physical properties of a fabric tube. The calculations were made with the aid of thin-walled cylinder theory.

Fabric Stress-Strain Requirements

On the assumption that the rubber bladder has negligible resistance to extension, the required stress-strain properties of the fabric were determined for the longitudinal direction, the circumferential direction, and for shearing. In the case of the longitudinal direction, the outside fiber of the bent tube was considered, and it was found that this direction requires the most unique mechanical behavior.

The important reason for obtaining the required stress-strain relations was so that the specifications could be written in terms of mechanical properties of fabrics.

This made it possible to analyze fabric structures without regard to tube radius or length.

Fabric Stress-Strain Results

In these results, I have included the requirements of bending and twisting so that the fabric results can be easily compared with the specifications. Also, to give an indication of how specialized the fabric structures must be, I have evaluated an elastic material that has been designed to meet the pressure requirements, and determined how it would behave in bending and twisting. Of course, these results are extremely high, and presented for comparison purposes only. (The shear stress in this calculation made use of the assumption that the shear modulus of rubber is about one-half to one-third the tensile modulus).

In the fabrics that were analyzed, one of two possibilities hold. Either the fabric could be designed to meet the specifications, or the fabric could be designed to come as close as possible to the specifications. In any event, each fabric has certain requirements of geometry or yarn properties that it must fulfill before it can yield the calculated results. The conflict that arises in selecting the best fabric is in the fact that in the fabrics that can be designed to meet the specifications, there is a possibility that the required geometry and physical properties cannot be obtained. In the fabrics that can easily fulfill their requirements, there is always a limiting factor which prevents meeting of the specifications.

Fabrics that can be designed to meet the bending specifications but have requirements that cannot definitely be achieved, include:

1. Diamond Fabric with Compression Resistant Yarns
2. Residual Stress Yarn Plain Weave Fabric
3. Residual Stress Diamond Fabric

4. Loop Element Fabric

For example, in the Daimond Fabric with Compression Resistant Yarns, if yarns can be produced with the desired properties in compression, then this fabric will meet the specifications. However, whether or not this can be done is not yet known.

The fabrics which can definitely be produced as desired, but which do not fully meet the requirements include:

1. Diamond Fabric with Slipping Intersections
2. Diamond Fabric with Rigid Intersections
3. Diamond Fabric with Elastic Yarns
4. Diamond Fabric with Series Yarns

The best of these fabrics is the Diamond Fabric with Slipping Intersections since it most nearly meets the bending and twisting stress-strain requirements. Since this fabric can definitely be made, I believe it should be tested before any of the fabrics in the first group. In the event its mechanical behavior is not satisfactory, the other fabrics of the second group should be tested; and if they fail, an attempt should be made to make the fabrics of the first group.

Results of Diamond Fabric Tube

Since the diamond fabric with slipping was found to be the best fabric, its behavior in a tube was analyzed. Results of bending moment vs. curvature and twisting moment vs. torsion were found for tubes of various radii. These results indicate an error of a factor of four in bending and a factor of 18 in twisting. Nevertheless, since many approximations were made in the analysis, and most of them such that they would give higher than actual results, I feel that there is a good chance that this fabric will prove successful. Since the error that arises in twisting is very serious, I have discussed the possibility of putting shear joints in the parts of the suit that require that type of movement. If they are used, it may be possible to use a fabric with rigid intersections.

It was found that the effect of tube radius is very great on the bending and twisting moments. There is a proportionality to the fourth power of the radius. That is, if the radius is decreased by a factor of one-half, the moment will decrease by a factor of one-sixteenth. It is therefore absolutely necessary that the tubes of which the pressure suit is made are the smallest possible size. It is also quite important that the instrument which is used to test the fabrics can test tubes of different radii.

PROPOSAL FOR FUTURE WORK

In this section, I will discuss the work that I think should follow this thesis for the purpose of creating a suitable pressure suit.

First, I believe that a careful study should be made to establish suitable specifications. This involves making measurements of curvature and torsion that are imposed on the suit as various parts of the body move. Also, by means of springs, or rubber testing devices, the allowable bending and twisting moments that can be applied comfortably by the body should be measured. Knowing the necessary tube size, the curvature and torsion imposed, and the allowable moments, a specification can be written for every tube that will be in the suit.

The next problem that should be solved is that of evaluating some of the fabrics that were theoretically designed. To do this, a measuring instrument must be made. The instrument should be able to measure bending moment vs. curvature, and twisting moment vs. torsion for different radii tubes that are under pressure. The ranges should be those that are encountered in the body. (Of course these must first be established as outlined in the preceding paragraph.)

If after testing the fabrics, it is found that it is impossible to meet the specifications, there are three methods which I believe should be investigated:

1. Use fabric structures which are originally in the shape of a bent cylinder with the amount of bend equal to half that which is required by the bending specification. The strains which will be imposed will be less than before; but, there will be a moment necessary to keep the tube straight.

2. Use shear joints as described on page 85. to get the required shear mobility.

3. Make the suit out of rubber and have it initially very much undersize. Then, develop a method by which pressure can be applied to the suit before it is put on, and have the wearer get into it when it is expanded up to size by the pressure. Once this suit is on it will be very flexible since it is made of rubber. Any amount of flexibility can be obtained by using the proper rubber and making it enough undersize. Of course, the big disadvantage of this type of suit is that it would be difficult to put on, and it can only be worn with the pressure on.

REFERENCES

Advanced Strength of Materials. J. P. den Hartog, New York: McGraw Hill Book Company, 1952

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Reference 2   Page 234